

Contents

Prologue	1
1 Fluid mechanics and the shock development problem	43
1.1 General equations of motion	43
1.2 The irrotational case and the nonlinear wave equation	54
1.3 The non-relativistic limit	58
1.4 Jump conditions	73
1.5 The shock development problem	87
1.6 The restricted shock development problem	97
2 Geometric construction	101
2.1 General construction in Lorentzian geometry	101
2.2 The characteristic system	107
2.3 The wave system	114
2.4 Variations by translations and the wave equation for the rectangular components of β	116
2.5 Geometric construction for the shock development problem	125
3 Acoustical structure equations	129
3.1 Connection coefficients and the first variation equations	129
3.2 Structure functions and the formulas for the torsion forms	135
3.3 Propagation equations for λ and $\underline{\lambda}$	147
3.4 Second variation and cross variation equations	148
3.5 The case $n = 2$	153
3.6 The Codazzi and Gauss equations ($n > 2$)	158
4 The problem of the free boundary	165
4.1 Analysis of the boundary conditions	165
4.2 Transformation functions and identification equations	171
4.3 Regularization of identification equations	193
5 Initial data and derived data	199
5.1 Propagation equations for $\underline{\lambda}$ and s_{NL} on \mathcal{C}	199
5.2 Propagation equations for higher-order derived data $T^m \underline{\lambda}$ and $T^m s_{NL}$ on \mathcal{C}	208
5.3 Boundary conditions for higher-order derived data and determination of the T -derivatives of the transformation functions on $\partial_- \mathcal{B}$. .	211

6	Variation fields	229
6.1	Bi-variational stress	229
6.2	Variation fields V and associated 1-forms $(^V)\theta^\mu$	233
6.3	Fundamental energy identities	239
6.4	Boundary condition on \mathcal{K} for the 1-forms $(^V)\xi$	246
7	Multiplier field	267
7.1	Coercivity at the boundary. Choice of multiplier field	267
7.2	Deformation tensor of the multiplier field. Error integral associated to $(^V)Q_1$	277
7.3	Error integral associated to $(^V)Q_2$	286
8	Commutation fields	297
8.1	Commutation fields and higher-order variations	297
8.2	Recursion formulas for source functions	298
8.3	Deformation tensors of the commutation fields	305
8.4	Principal acoustical error terms	309
9	Power series approximation method	319
9.1	Setup of the truncated power series	319
9.2	Estimates for the quantities by which the N th approximants fail to satisfy the characteristic and wave systems	322
9.3	Estimates for the quantities by which the N th approximants fail to satisfy the boundary conditions	337
9.4	Estimates for the quantities by which the N th approximants fail to satisfy the identification equations	345
9.5	Estimates for the quantity by which the $\beta_{\mu,N}$ fail to satisfy the wave equation relative to \tilde{h}_N and to \tilde{h}'_N	348
9.6	Variation differences $(^{m,l})\check{\phi}_\mu$ and rescaled source differences $(^{m,l})\check{\rho}_\mu$	362
9.7	Difference 1-forms $(^V; m, l)\check{\xi}$. Difference energies and difference energy identities	368
10	Top-order acoustical estimates in the case $d = 2$	373
10.1	Regularization of the propagation equations for $\check{\chi}$ and $\underline{\check{\chi}}$	373
10.2	Regularization of the propagation equations for $E^2 \lambda$ and $E^2 \underline{\lambda}$	379
10.3	Structure equations for the N th approximants	390
10.4	Propagation equations for $\check{\theta}_l$, $\underline{\check{\theta}}_l$ and for $\check{v}_{m,l}$, $\underline{\check{v}}_{m,l}$	408
10.5	Estimates for $\check{\theta}_l$	426
10.6	Estimates for $\check{v}_{m-1,l+1}$	448
10.7	Estimates for $\check{\theta}_l$ and $\underline{\check{v}}_{m-1,l+1}$ in terms of their boundary values on \mathcal{K}	471

10.8	Boundary conditions on \mathcal{K} and preliminary estimates for $\check{\underline{\gamma}}_l$ and $\check{\underline{\gamma}}_{m-1,l+1}$ on \mathcal{K}	498
11	Outline of top-order acoustical estimates for more than 2 spatial dimensions	513
12	Top-order estimates for transformation functions and next-to-top-order acoustical estimates	543
12.1	Propagation equations for the next-to-top-order acoustical difference quantities $(^{(n-1)}\check{\chi}, ^{(n-1)}\check{\underline{\chi}})$ and $(^{(m,n-m)}\check{\lambda}, ^{(m,n-m)}\check{\underline{\lambda}}) : m = 0, \dots, n$	543
12.2	Estimates for $(^{(n-1)}\check{\chi}, ^{(n-1)}\check{\underline{\chi}})$ and $(^{(0,n)}\check{\lambda}, ^{(0,n)}\check{\underline{\lambda}})$	566
12.3	Estimates for $(T\Omega^n \check{f}, T\Omega^n \check{v}, T\Omega^n \check{y})$	635
12.4	Estimates for $(^{(m,n-m)}\check{\lambda}, ^{(m,n-m)}\check{\underline{\lambda}}) : m = 1, \dots, n$	649
12.5	Estimates for $(T^{m+1} \Omega^{n-m} \check{f}, T^{m+1} \Omega^{n-m} \check{v}, T^{m+1} \Omega^{n-m} \check{y}) : m = 1, \dots, n$	706
12.6	Estimates for $(\Omega^{n+1} \check{f}, \Omega^{n+1} \check{v}, \Omega^{n+1} \check{y})$	720
13	Top-order energy estimates	741
13.1	Estimates for $^{(V;m,n-m)}\check{b}$	741
13.2	Borderline error integrals contributed by $^{(V;m,n-m)}\check{Q}_1, ^{(V;m,n-m)}\check{Q}_2$	754
13.3	Borderline error integrals associated to $\check{\theta}_n$ and to $\check{\underline{\gamma}}_{m-1,n-m+1} : m = 1, \dots, n$	755
13.4	Borderline error integrals associated to $\check{\theta}_n$ and to $\check{\underline{\gamma}}_{m-1,n-m+1} : m = 1, \dots, n$	765
13.5	Top-order energy estimates	777
14	Lower-order estimates, recovery of the bootstrap assumptions, and completion of the argument	791
14.1	Summary of the preceding, outline of the following, and statement of the theorem	791
14.2	Bootstrap assumptions needed	797
14.3	$L^2(S_{\underline{u},u})$ estimates for $^{(n-1)}\check{\chi}$ and for $^{(m,n-m)}\check{\lambda} : m = 0, \dots, n-1$	806
14.4	$L^2(\mathcal{K}_\sigma^\tau)$ estimates for the n th-order acoustical differences	819
14.5	$L^2(S_{\underline{u},u})$ estimates for the n th-order variation differences	828
14.6	$L^2(S_{\underline{u},u})$ estimates for the $(n-1)$ th-order acoustical differences	842
14.7	$L^2(S_{\underline{u},u})$ estimates for all n th-order derivatives of the β_μ	851
14.8	$L^2(S_{\underline{u},u})$ estimates for $\Omega^{n-1} \log \#$ and $\Omega^{n-1} b$	855
14.9	Lower-order $L^2(S_{\underline{u},u})$ estimates	860
14.10	Pointwise estimates and recovery of the bootstrap assumptions	863
14.11	Completion of the argument	869
	Bibliography	915
	Index	917