Introduction to Part I

This book begins with an introduction to flows—as opposed to hyperbolic flows, to which the second part is dedicated. While this serves as a self-contained introduction to the basic concepts in dynamics, readers familiar with dynamics in discrete time will see this as a parallel development in which the distinctions between dynamics in continuous time and discrete time begin to reveal themselves. And while we treat flows in rather great generality here, the selection of notions and phenomena we include is informed by those we will later be able to observe in and apply to hyperbolic flows.

Chapter 1 introduces flows from a topological point of view and after introducing the most basic notions explores recurrence ideas all the way to Conley's Fundamental Theorem of Dynamical Systems, the existence of dense orbits and (topological) mixing properties, concluding with a strong notion of sensitive dependence on initial conditions (expansivity) and symbolic flows, both of which are central to hyperbolic dynamical systems. While this chapter is replete with examples, the optional Chapter 2 introduces a foundational example in hyperbolic dynamics, the geodesic flow of a surface of constant negative curvature. This treatment is elementary and explicit but foreshadows features, and to some extent methods, that in Part II will be seen as characteristic of hyperbolic flows. Section 5.2 can be seen as a direct follow-up. More broadly, Chapters 5 and 6 build on these chapters.

Chapter 3 is a probabilistic counterpart to Chapter 1. It introduces measurable and measure-preserving flows and the pertinent notions and results from ergodic theory. This development and that in the next chapter are supported by Appendix A which introduces the basic notions regarding probability and Lebesgue spaces as well as the discrete-time counterparts to the theory developed here. Chapter 4 continues and connects the developments in both Chapters 1 and 3 by presenting the core of entropy theory, which, being centered on notions of exponential orbit complexity, is a natural and important tool in hyperbolic dynamics. Indeed, the notions from these chapters will be applied in Chapter 7.