Introduction to Part II

We now come to the principal subject matter of this book, hyperbolic dynamics. The next three chapters contain the general theory of hyperbolic flows, and these chapters develop topological, measurable, and differentiable properties for these flows. The last two chapters of this section investigate properties of Anosov flows, and these two chapters contain numerous recent results.

Chapter 5 defines hyperbolicity and develops its essential features, as well as a range of new examples—several of which have not previously appeared outside the research literature. Many of the results in this chapter are actually consequences of two properties of hyperbolicity—expansivity and the shadowing property. Following a strategy of Anosov, Katok, and Bowen, we make a point in this chapter to highlight the number of results that can be obtained from just these two aspects of hyperbolic flows. Chapter 6 introduces the concept of stable and unstable manifolds. The resulting foliation structure refines our understanding of hyperbolic dynamics. Chapter 7 studies measurable and statistical aspects of hyperbolic flows; hyperbolic dynamics is deterministic but of such complexity that a probabilistic approach is natural. Related regularity issues are refined in the chapter on rigidity (Chapter 9).

The concluding chapters are (mainly) dedicated to Anosov flows, and we pursue two topics further. A topological study (Chapter 8) explores dynamical and structural features of Anosov flows as well as new examples of them. Chapter 9 is more focused on smooth dynamics and explores among other things a range of situations in which the generally rare circumstance of smooth conjugacy (or orbit equivalence) arises in natural contexts from the coincidence of some dynamical features with those of an algebraic counterpart. Most of the results in these last two chapters come with a proof, but unlike the previous parts of this book we do not strive to prove all the results we use. At the least we provide references where the proofs can be found.