Preface

This text grew from notes for a graduate course of thirty hours that I gave at the University of Roma "Tor Vergata" in the academic year 2018–19. The subject is very classical, i.e., the classification of complex algebraic surfaces, which goes back to the old Italian school of algebraic geometry with Enriques and Castelnuovo. However, the exposition is modern since it follows Mori's approach to the classification of algebraic varieties. The text includes the P_{12}-theorem, the Sarkisov programme in the surface case and the Noether–Castelnuovo theorem in its classical version. I am indebted to the first chapter of Matzuki's book [17], though I simplified and streamlined his exposition at some points. For the P_{12}-theorem I partly followed the unpublished paper [10] by Francia and the recent paper [7] by Catanese–Li. The classification of bielliptic surfaces follows the short and elegant exposition by Bombieri–Mumford in [5]. The classical version of the Noether–Castelnuovo theorem follows Calabri's notes [6].

This text is intended for those with good knowledge of basic algebraic geometry. Generally speaking, acquaintance with basic parts of books like [12, 13] should be sufficient. In any case I have tried to make the text as self-contained as possible. For this reason, a first chapter is devoted to a quick exposition of some preliminaries.