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Sebastian Baader

Geometry and Topology of Surfaces

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For Alessandra Iozzi

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The chapter on the Basmajian identity may come as a surprise, and would not exist without Hugo Parlier, who explained it to me last summer in Les Diablerets. Thank you for that, Hugo.

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