Preface

This manuscript was originally conceived as the lecture notes for an introductory graduate-level course taught by Francisco Marcellán during the First Orthonet Summer School in Seville, Spain, 2016. The purpose of the course (and by transitivity, the purpose of these notes as well) was to introduce young researchers to the basic theory of orthogonal polynomials using linear functionals as the main tool for treating several notions of the theory.

Since these lecture notes were first written, we have gradually added several results about orthogonal polynomials obtained by working with linear functionals that appear scattered throughout the literature and that we thought would nicely fit in with the contents of the original notes. Our intention was to prepare a document that students and interested researchers can consult as an introductory text to this branch of study. Moreover, most of these results appear in regular scientific journals and their proofs are tailored for a more mature audience. Thus, in the spirit of making this manuscript a point of first approach to the field, we have filled in some of the details in the proofs of a few results whenever we thought appropriate to do so.

We must say that these notes are far from being an exhaustive account of the development of the general theory of orthogonal polynomials. We adhere to describing results concerning standard orthogonality with respect to linear functionals. Nevertheless, other types of orthogonality are widely studied as well. For instance, orthogonal polynomials with respect to the so-called Sobolev bilinear forms have been of great interest in recent years. This type of orthogonality is quite different from that associated with linear functionals, and the properties of the corresponding orthogonal polynomials deviate greatly from those presented in these notes. Hoping to make up for this lack of exhaustiveness, we have added a list of references that an interested reader can consult.