

# Contents

<b>1</b>	<b>Some model equations</b>	1
1.1	The Yamabe equation . . . . .	1
1.2	The KGMP and SP systems . . . . .	4
1.3	The Einstein-scalar field Lichnerowicz equation . . . . .	10
<b>2</b>	<b>Basic variational methods</b>	14
2.1	Some notation and basic facts . . . . .	14
2.2	The variational method by minimization . . . . .	15
2.3	The variational method based on the mountain pass lemma . . . . .	21
2.4	A few words on the Einstein-scalar field Lichnerowicz equation . . . . .	27
2.5	Solving critical equations.1 . . . . .	30
2.6	Playing with symmetries – The case of large potentials . . . . .	37
2.7	Solving critical equations.2 . . . . .	38
2.8	Regularity theory . . . . .	42
<b>3</b>	<b>The <math>L^p</math> and <math>H^1</math>-theories for blow-up</b>	48
3.1	The $L^p$ -theory for blow-up . . . . .	49
3.2	The $H^1$ -theory for blow-up . . . . .	54
3.3	Proof of Theorem 3.3 . . . . .	58
3.4	Proof of Lemma 3.5 . . . . .	63
3.5	Remarks on Theorem 3.3 . . . . .	76
<b>4</b>	<b>Blowing-up solutions in the critical case</b>	83
4.1	The sphere model case . . . . .	83
4.2	Variations on the above theme . . . . .	86
4.3	Infinite energy solutions.1 . . . . .	92
4.4	The low-dimensional case . . . . .	96
4.5	Weakly critical versus critical potentials . . . . .	99
4.6	The finite dimensional reduction method in few words . . . . .	102
4.7	Blowing-up solutions in arbitrary manifolds . . . . .	104
4.8	Infinite energy solutions.2 . . . . .	108
4.9	The Yamabe equation in high dimensions . . . . .	109
4.10	Blow-up type configurations . . . . .	110
4.11	The model Equations . . . . .	111
<b>5</b>	<b>An introduction to elliptic stability</b>	113
5.1	A first insight into elliptic stability . . . . .	114
5.2	Stability and standing waves for NLS and NKG . . . . .	119
5.3	The subcritical case of stationary Schrödinger's equations . . . . .	120

5.4	Various notions of stability in the critical case . . . . .	122
5.5	The supinf 3-dimensional inequality . . . . .	125
<b>6</b>	<b>Bounded stability</b>	129
6.1	Blow-up theory in the one-bubble model case . . . . .	131
6.2	A Riemannian version of the Pohozaev identity . . . . .	147
6.3	Blow-up theory in the one-bubble model case (continued) . . . . .	149
6.4	Proof of Theorem 6.1 . . . . .	162
6.5	Proof of Theorem 6.3 . . . . .	173
6.6	The Brézis-Li uniqueness result . . . . .	175
6.7	Compactness for the Yamabe equation and Theorem 6.2 . . . . .	177
<b>7</b>	<b>The <math>C^0</math>-theory for blow-up</b>	180
7.1	A first set of pointwise estimates . . . . .	182
7.2	Proof of the upper estimate in Theorem 7.1 . . . . .	198
7.3	Basic computations . . . . .	229
7.4	Proof of Theorem 7.2 and of the lower estimate in Theorem 7.1 . . . . .	237
7.5	Coercivity is a necessary assumption . . . . .	243
<b>8</b>	<b>Analytic stability</b>	245
8.1	Proof of Theorems 8.1 and 8.2 in the conformally flat case . . . . .	247
8.2	The range of influence of blow-up points . . . . .	255
8.3	Proof of Theorems 8.1 and 8.2 in the general case . . . . .	266
8.4	Blow-up in the 6-dimensional case . . . . .	279
8.5	The model Equations . . . . .	281
	<b>Bibliography</b>	285