Contents

1 Introduction 5
- 1.1 Motivation for cluster algebras . . . 5
- 1.2 Some recurrences . . . 7
- 1.3 Somos recurrences . . . 8
- 1.4 Why study cluster algebras? . . . 9
- 1.5 Some notation . . . 9

2 Cluster algebras 10
- 2.1 Definition of a cluster algebra . . . 10
- 2.2 Skew-symmetrizable matrices . . . 13
- 2.3 Quiver notation . . . 13
- 2.4 Valued quiver notation . . . 16
- 2.5 Exchange graphs . . . 17

3 Exchange pattern cluster algebras 19
- 3.1 Exchange patterns . . . 19
- 3.2 Exchange pattern cluster algebras . . . 22
- 3.3 Matrices of exponents . . . 23

4 Reflection groups 31
- 4.1 Definition of a reflection group . . . 31
- 4.2 Root systems . . . 33
- 4.3 Simple systems . . . 35
- 4.4 Coxeter groups . . . 36
- 4.5 Crystallographic root systems and Cartan matrices . . . 37
- 4.6 Classification of finite crystallographic reflection groups . . . 39
- 4.7 Type A_n . . . 39
- 4.8 Root systems of low rank . . . 41
- 4.9 Finite Coxeter groups . . . 43
- 4.10 Reduced expressions . . . 44
- 4.11 Coxeter elements . . . 47

5 Cluster algebras of finite type 49
- 5.1 Classification . . . 49
- 5.2 Folding . . . 53
- 5.3 Denominators . . . 55
- 5.4 Root clusters . . . 58
- 5.5 Admissible sequences of sinks . . . 59
- 5.6 Number of clusters . . . 62

6 Generalized Associahedra 63
6.1 Fans . . . 63
6.2 The cluster fan . . . 64
6.3 The cluster complex . . . 67
6.4 Normal fans . . . 69
6.5 The generalized associahedron . . . 70
6.6 The generalized associahedron of type A_n . . . 71

7 Periodicity 75
7.1 The Somos-4 recurrence . . . 75
7.2 Period 1 quivers . . . 76
7.3 Periodicity in the coefficient case . . . 78
7.4 Higher period quivers . . . 80
7.5 Categorical periodicity . . . 80

8 Quivers of finite mutation type 84
8.1 Classification . . . 84
8.2 Tagged triangulations . . . 87

9 Grassmannians 90
9.1 Exterior powers . . . 90
9.2 The Grassmannian . . . 91
9.3 The Grassmannian $Gr(2,n)$ . . . 92
9.4 The Grassmannian $Gr(k,n)$ . . . 94
9.5 The Grassmannian $Gr(2,n)$ revisited . . . 98

Bibliography 101

Nomenclature 111

Index 115