

Contents

I	Introduction	1
1	The low density limit	2
1.1	The Liouville equation	3
1.2	Mean field versus collisional dynamics	4
1.3	The Boltzmann-Grad limit	6
2	The Boltzmann equation	7
2.1	Transport and collisions	7
2.2	Boltzmann's H-theorem and irreversibility	8
2.3	The Cauchy problem	10
2.3.1	Short-time existence of continuous solutions.	10
2.3.2	Fluctuations around some global equilibrium.	11
2.3.3	Renormalized solutions.	12
3	Main results	14
3.1	Lanford and King's theorems	14
3.2	Background and references	15
3.3	New contributions	17
II	The case of hard spheres	19
4	Microscopic dynamics and BBGKY hierarchy	20
4.1	The N -particle flow	20
4.2	The Liouville equation and the BBGKY hierarchy	22
4.3	Weak formulation of Liouville's equation	23
4.4	The Boltzmann hierarchy and the Boltzmann equation	27
5	Uniform a priori estimates for the BBGKY and Boltzmann hierarchies	30
5.1	Functional spaces and statement of the results	30
5.2	Main steps of the proofs	33
5.3	Continuity estimates	34
5.4	Some remarks on the strategy of proof	38
6	Statement of the convergence result	39
6.1	Quasi-independence	39
6.1.1	Admissible Boltzmann data.	39
6.1.2	Conditioning.	40
6.1.3	Characterization of admissible Boltzmann initial data.	41

6.2 Main result:	
Convergence of the BBGKY hierarchy to the Boltzmann hierarchy	46
6.2.1 Statement of the result.	46
6.2.2 About the proof of Theorem 8: outline of Chapter 7 and Part IV.	47
7 Strategy of the proof of convergence	49
7.1 Reduction to a finite number of collision times	50
7.2 Energy truncation	51
7.3 Time separation	53
7.4 Reformulation in terms of pseudo-trajectories	54
III The case of short-range potentials	57
8 Two-particle interactions	58
8.1 Reduced motion	58
8.2 Scattering map	61
8.3 Scattering cross-section and the Boltzmann collision operator	64
8.3.1 Scattering cross-section.	64
8.3.2 Boltzmann collision operator.	67
9 Truncated marginals and the BBGKY hierarchy	68
9.1 Truncated marginals	69
9.2 Weak formulation of Liouville’s equation	70
9.3 Clusters	74
9.4 Collision operators	76
9.5 Mild solutions	78
9.6 The limiting Boltzmann hierarchy	79
10 Cluster estimates and uniform a priori estimates	81
10.1 Cluster estimates	81
10.2 Functional spaces	84
10.3 Continuity estimates	86
10.4 Uniform bounds for the BBGKY and Boltzmann hierarchies	90
11 Convergence result and strategy of proof	91
11.1 Admissible initial data	91
11.2 Convergence to the Boltzmann hierarchy	93
11.3 Reductions of the BBGKY hierarchy, and pseudo-trajectories	94
IV Termwise convergence	98
12 Elimination of recollisions	99
12.1 Stability of good configurations by adjunction of collisional particles	99
12.2 Geometric lemmas	102

12.2.1	Bad trajectories associated to free transport.	102
12.2.2	Modification of bad trajectories by hard sphere reflection.	103
12.2.3	Modification of bad trajectories by the scattering associated to Φ	103
12.3	Proof of the geometric proposition	104
12.3.1	The pre-collisional case.	104
12.3.2	The post-collisional case with hard sphere reflection.	105
12.3.3	The post-collisional case with smooth scattering.	106
13	Truncated collision integrals	109
13.1	Initialization	109
13.2	Approximation of the Boltzmann functional	110
13.3	Approximation of the BBGKY functional	113
14	Proof of convergence	117
14.1	Proximity of Boltzmann and BBGKY trajectories	117
14.2	Proof of convergence for the hard-sphere dynamics: proof of Theorem 8	121
14.2.1	Error coming from the initial data.	122
14.2.2	Error coming from the prefactors in the collision operators.	124
14.2.3	Error coming from the divergence of trajectories.	124
14.2.4	Optimization for tensorized Lipschitz initial data.	125
14.3	Convergence in the case of a smooth interaction potential: proof of Theorem 11	126
15	Concluding remarks	128
15.1	On the time interval of validity of Theorems 8 and 9	128
15.2	More general potentials	128
15.3	Other boundary conditions	129
Bibliography		130
Index		137