Preface

In the early decades of the 20th century, the objective of class field theory was to relate the arithmetic of a given finite extension field L of an algebraic number field K with abelian Galois group to the properties of the base field K. Two fundamental lines of research regarding this problem were intimately interwoven with one another, one being the quest for a law of decomposition in L for the prime ideals in K, the other being the wish to obtain a general reciprocity law, similar to the law of Carl Friedrich Gauss in the case of a quadratic extension L/K. In 1927, Emil Artin's reciprocity law provided the decisive link between these two theories. It is, in its abstract form, the heart of the theory of abelian extensions of algebraic number fields. However, his work relied to a large extent on the study of analytic invariants such as ζ-functions or L-functions attached to algebraic number fields.

At that time, Artin, born in Vienna on March 3, 1898, was already Ordinarius at the recently founded University of Hamburg. He had completed his Ph.D. in Leipzig with Gustav Herglotz in 1921 and his Habilitation in Hamburg in 1923, and was finally appointed as Ordentlicher Professor in 1926. In the following years Artin was, together with his colleagues Erich Hecke and Wilhelm Blaschke, a most active member of the Mathematisches Seminar der Universität Hamburg. His works in various areas in number theory, algebra and topology during that time earned him a reputation as a distinguished scholar who was held in high esteem in the scientific community as both teacher and researcher.

Unfortunately, however, the political events in Germany would disrupt Artin's progress and plans. The German government prevented Emil Artin from attending the International Congress of Mathematicians in Oslo, Norway, in 1936 and refused to grant Artin permission to deliver a series of lectures at Stanford University in the U.S.A. in 1937. These events were consequences of the dramatic change in the political situation in Germany in January 1933 when Adolf Hitler and the Nazi party had assumed control of Germany.

"It was only a question of time", Richard Brauer would later describe it, "until [Emil] Artin, with his feeling for individual freedom, his sense of justice, his abhorrence of physical violence would leave Germany".[1] By the time Hitler issued the edict on January 26, 1937, that removed any employee married to a Jew from their position as of July 1, 1937,[2] Artin had already begun to make plans to leave Germany. Artin had married his former student, Natalia Jasny, in 1929, and, since her father practiced the Jewish faith, the Nazis classified her as Jewish.[3] On October 1, 1937, Artin and his family arrived in America.

[1] Brauer, Richard, Emil Artin, *Bulletin of the American Mathematical Society* 73 (1967), p. 28.

[2] Art. 59 in the "Neues Deutsches Beamtengesetz" (New German Civil Service Law), later on supplemented by the so-called "Flaggenerlass" of April 19, 1937.

[3] Already on September 27, 1934, Artin had to declare in an official statement that his wife was of "non-Aryan descent", see Reich, Karin, Emil Artin – Mathematiker von Weltruf, in *Das Hauptgebäude der Universität Hamburg als Gedächtnisort*, edited by Rainer Nicolaysen, Hamburg University Press, Hamburg 2011, p. 57.

These moments in Emil Artin's life naturally raise some compelling questions. How did class field theory develop in the 1930s? How did Artin's contributions influence other mathematicians at the time and in subsequent years? Given the difficult political climate and his forced emigration as it were, how did Artin create a life in America within the existing institutional framework? Did Artin continue his education of and close connection with graduate students? Finally, is it possible to begin to come to terms with the influence of Artin on present day work on a non-abelian class field theory?

Our attempt to investigate these questions led to individual essays by the authors and two contributors, James Cogdell and Robert Langlands, that now form the chapters of this volume.[4] Like Carl Schorske's compilation of writings for his celebrated study of *fin-de-siècle* Vienna, each chapter "issued from a separate foray into the terrain, varying in scale and focus according to the nature of the problem".[5] Taken together, these chapters offer a view of both the life of Artin in the 1930s and 1940s and the development of class field theory at that time. They also provide insight into the transmission of mathematical ideas, the careful steps required to preserve a life in mathematics at a difficult moment in history, and the interplay between mathematics and politics (in more ways than one). Some of the technical points in this volume require a sophisticated understanding of algebra and number theory. The broader topics, however, will appeal to a wider audience that extends beyond mathematicians and historians of mathematics to include historically minded individuals, particularly those with an interest in the time period.

We take Claude Chevalley's presence in Artin's 1931 Hamburg lectures on the development of class field theory as our starting point. We consider Chevalley's earlier work in class field theory, his thesis, and Artin's early influence on these mathematical developments. We give especial attention to Chevalley's 1935 letter to Helmut Hasse where he presents the concept of *éléments idéaux*. We then turn our attention to the intersection of Artin's personal and professional lives as he made his way to the U.S. in 1937 through the time of his appointment at Princeton in 1946. Specifically, we explore the behind-the-scenes initiatives to secure Artin his first temporary position at the University of Notre Dame and his move to a permanent position at Indiana University in 1938 where he worked with George Whaples on the foundations of algebraic number theory. We introduce the reader to Margaret Matchett, Artin's Ph. D. student at Indiana, and her thesis "On the zeta function for ideles". The book also considers the influence of Artin on contemporary work on non-abelian class field theory as found in the seminal letter of Robert Langlands to André Weil. In this epistolary conversation, Langlands describes his general concept of Euler products, incorporating the known L-series of Hecke and Artin in one notion. In his chapter on L-functions, James Cogdell provides the bridge between Artin's L-functions as

[4] In preparing Chapters II and III, we have drawn from past collaborative and individual sources [64], [60], and [59].

[5] Schorske, Carl E., *Fin-de-Siècle Vienna*, Alfred A. Knopf, New York, 1980, p. xxviii.

they appear in his general reciprocity law and the automorphic L-functions encoded in Langlands' letter. The book concludes with Langlands' personal commentary on his letter to Weil, revealing his early mathematical thoughts and challenges.

The first chapter, "Class field theory: From Artin's course in Hamburg to Chevalley's 'Éléments idéaux' ", presents, for the first time, the letter from Claude Chevalley to Helmut Hasse outlining his notion of *éléments idéaux*. It also provides the historical context for these ideas and, in so doing, exposes the journey not only of an idea but also of a mathematician. Chevalley attended Artin's course on class field theory in 1931 in Hamburg. He submitted his thesis "Sur la théorie du corps de classes dans les corps finis et le corps locaux" in 1932. By 1935, Chevalley had been invited by Hasse and Erich Hecke, in their new roles as editors of Volume I "Algebra und Zahlentheorie" of the *Encyklopädie der Mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, to contribute an article on class field theory. In his seminal publications of 1936 and 1940 on this topic, Chevalley replaced classical ideal-theoretic approaches to number theory with his "les éléments idéaux", or as it was called in 1940, "notion de idèle". In less than a decade, Chevalley progressed from student to specialist on class field theory. In terms of geography, Chevalley traveled from France to Germany to the United States, interacting with Japanese mathematicians along the way, calling attention to a relatively new international phenomenon in the life of a mathematician.

Chapter II, "Creating a life: Emil Artin in America", acquaints the reader with Artin at the moment when political turbulence affected his life in a real and tangible way. The news events that splashed across the top of newspapers were more than headlines for Artin; they were vivid historical events that manifested themselves in the moments of his life, with his family and with his mathematics. In his role as President of the American Mathematical Society, Solomon Lefschetz was particularly "well placed to know what was going on" relative to worldwide political events and the subsequent needs of mathematicians, in this case, the needs of Emil Artin and his family. On January 12, 1937, Lefschetz wrote to Father O'Hara to urge him to consider a position for Emil Artin on his faculty at University of Notre Dame. After one year at Notre Dame, Artin moved to a permanent position at Indiana University where, it seems, the teaching grounded him. The steady, methodical rhythm of the daily life of imparting mathematics to fresh faces established a routine for him, a pattern not so unlike what he searched for in mathematics.

At the same time, the teaching helped restore a sense of balance to Artin. It extended him, in the form of George Whaples and Margaret Matchett who gave his larger ideas – those beyond trigonometry and calculus, for example – a chance to grow and develop into research level mathematics.

Chapter III focuses on Artin's work with Whaples, which was very much inspired by Chevalley's notion of the idele group. Whaples came to Indiana by way of the University of Wisconsin, where he earned his Ph.D., under the direction of Mark H. Ingraham, with a thesis "On the structure of modules with a commutative algebra as operator domain". Since Ingraham studied with E. H. Moore at the University of Chicago, Whaples enjoyed a distinctly American heritage in his mathematical training. Artin and Whaples defined valuation vectors as the additive counterpart of the group of ideles attached to an algebraic number field. This association enabled them to derive the fundamental results of number theory from simple axioms. Their main result is the use of the product formula for valuations to derive an axiomatic characterization of both algebraic number fields and function fields with a finite field of constants. These are exactly the two families of fields for which class field theory is known to hold.

Whaples' work and his association with Artin opened the door for him to visit the Institute for Advanced Study in Princeton. His application, an astonishingly concise, handwritten document reveals Whaples' summary and analysis of this work as well as his future research plans.

Chapter IV introduces the reader to Margaret Matchett, Artin's second Ph. D. student at Indiana, and her thesis "On the zeta function for ideles". While Matchett pursued her Ph. D. work, Artin and Whaples used the theory of valuations to investigate classical questions in algebraic number theory. It was only natural, then, for Artin to suggest to Matchett that she consider the possibility of interpreting the theory of zeta functions and L-series through the lens of ideles and valuation vectors. Her Ph. D. in 1946 was the only Ph. D. in mathematics awarded to a woman at Indiana in the 1940s. Exploring her life more generally provides further insights into broader trends in employment for women and personal ramifications of political ideologies.

In Chapter V, in his contribution "L-functions and non-abelian class field theory, from Artin to Langlands", James Cogdell rather poetically begins his study with Weil's assertion that Artin had a "love affair with the zeta function" while he was at Hamburg. In fact, it seems, Artin had a love affair with the zeta function throughout most of his mathematical life. Cogdell follows Chevalley's suggestion that Artin made use of zeta functions to discover precise algebraic facts rather than estimates or approximations. In particular, Cogdell argues that Artin used L-functions as a tool to study non-abelian class field theory. His chapter in this volume begins with an overview of L-functions before Artin, then follows the course of Artin's L-functions and the parallel development of the L-functions of Hecke and their "reconciliation" in the Langlands program.

That brings us to the final chapter of the volume, "Automorphic L-functions", contributed by Robert P. Langlands. In this chapter, we present the letter from Langlands to André Weil outlining his ideas concerning the definition of a new class of Euler products. Right at the beginning of the letter, the new concept of what is now known as the L-group appears in a quite sophisticated form. The theory of automorphic forms, in particular, the theory of Eisenstein series in the context of general reductive groups, serves as a source of inspiration for this circle of ideas. In addition to this letter, in the treatise "Funktorialität in der Theorie der automorphen Formen: Ihre Entdeckung und ihre Ziele", Langlands describes how his letter to Weil came into existence, comments on its mathematical content, and provides personal and professional insights about how his ideas unfolded, in particular, within the theory of automorphic forms. In his more personal remarks, Langlands describes his mathematical youth when he read Courant and Dickson; his desire to learn too fast; his frustration with a year that was not particularly fruitful in terms of mathematical results; his thoughts about giving up mathematics; and his success during the Christmas break of 1966–67, in his office in Fine Hall at Princeton, when he found the idea to prove the analytic continuation of automorphic L-functions. The ebb and flow of Langlands' professional life not only offers guideposts for a career in mathematics but also inspiration. Quite unexpectedly, then, this volume, that was originally intended to shed light on the development of class field theory as it passed through the hands of Chevalley, Artin and Langlands, grew into a rich exposition on the arc of a life in mathematics.

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Della Dumbaugh
Joachim Schwermer