General conventions

As said in the Introduction, this book is devoted to real and complex geometry; throughout it, k will denote a field (the *base field*) assumed to be either the real or the complex one. Unless otherwise said, k will remain fixed and all projective spaces, affine spaces and vector spaces will be assumed to be over k. The elements of the base field k will be called *scalars*, and sometimes also *constants*.

It is worth noting though, that with the only exceptions of Corollaries 2.4.3 and 2.4.4 (which require an infinite base field), the contents of Chapters 1 to 4 hold without changes if k is assumed to be a field with characteristic different from 2 (that is, with $1 + 1 \neq 0$) and other than the field $\mathbb{F}_3$ with three elements. Thus, for the first four chapters and with the above quoted exceptions, the reader may adopt this more general hypothesis as well.

As usual, $\mathbb{R}$ and $\mathbb{C}$ will denote the fields of the real and complex numbers, respectively, and $\mathbb{R}^+$ the set of all positive real numbers. The imaginary unit is denoted by $\boldsymbol{i}$, as the ordinary i is often used for other purposes. The identity map on a set X is written Id_X.

Usually, the entry in row i and column j of a matrix will be written in the form a^i_j, and also $a_{i,j}$ if the matrix is symmetric. The unit n-dimensional matrix will be denoted by $\mathbf{1}_n$, or just by $\mathbf{1}$ if no reference to the dimension is needed.

If A is any set, a subset $\{a, a'\} \subset A$ will be sometimes referred to as the *unordered pair* or just the *pair* composed by a, a'. This includes the case $a = a'$, in which we still consider $\{a\}$ as a pair: it will be called a *pair of coincident* or *repeated* elements of A, and often written $\{a, a\}$.

We will refer to the highest degree monomial (resp. coefficient) of a non-zero polynomial in one variable as its *leading monomial* (resp. *leading coefficient*). The polynomials which are non-zero and have leading coefficient equal to one are called *monic*. Greatest common divisors, minimal common multiples and irreducible factors of polynomials are always assumed to be monic. The polynomial 0 will be taken as a homogeneous polynomial of degree m for any non-negative integer m.

Unless otherwise stated, the roots of polynomials will be counted according to their multiplicities, that is, a root a of a polynomial $P(X)$ will be counted as many times as the number of factors $X - a$ appearing in the decomposition of $P(X)$ in irreducible factors.

Trigonometric functions will be taken as defined in function theory, for instance $\cos x = (e^{\boldsymbol{i}x} + e^{-\boldsymbol{i}x})/2$, and not by their relations to the elements of a right triangle, as the latter may or may not make sense or hold depending on the geometric context.

Many results of plane projective geometry give rise to graphic constructions which may be performed using a straight edge and, sometimes, a compass. We will make occasional references to some of these constructions, as they illustrate very well the related theory, but we do not intend to present them systematically. When dealing with graphic constructions the base field will be implicitly assumed to be the real one.