Preface

These notes deal with tempered homogeneous spaces $\overset{*}{A}{}^s_{p,q}(\mathbb{R}^n)$, $A \in \{B,F\}$, in the framework of the dual pairing $\big(S(\mathbb{R}^n), S'(\mathbb{R}^n)\big)$, where p,q,s are restricted to the distinguished strip

$$0 < p,q \le \infty, \qquad n\Big(\frac{1}{p} - 1\Big) < s < \frac{n}{p}.$$

Properties of their better known inhomogeneous counterparts $A^s_{p,q}(\mathbb{R}^n)$ can be transferred to $\overset{*}{A}{}^s_{p,q}(\mathbb{R}^n)$ as appropriate. Theorems 3.3, 3.5, 3.11, 3.20 and, in particular, 3.24 may be considered our basic assertions, on which we then rely to prove some specific properties. A characteristic feature of these notes is the careful distinction between several types of norms as explained in Section 1.3 (admissible, domestic, regional, community norms) and their use. Our motivation to study tempered homogeneous spaces $\overset{*}{A}{}^s_{p,q}(\mathbb{R}^n)$ comes from the Navier–Stokes equations. This is described in Section 1.1, but will not be used later on and can be skipped. In Sections 2.1–2.5, Chapter 2 collects (both more- and lesser-known) definitions and properties of the homogeneous spaces $\dot{A}^s_{p,q}(\mathbb{R}^n)$ in the framework of the dual pairing $\big(\dot{S}(\mathbb{R}^n), \dot{S}'(\mathbb{R}^n)\big)$. This is complemented in Section 2.6 by sketchy proposals to deal with further types of tempered homogeneous spaces (anisotropic spaces, hybrid spaces, spaces with dominating mixed smoothness, weighted spaces, radial spaces) following the set-up of these notes. The heart of these notes is Chapter 3, as indicated above. We also discuss what happens outside the above distinguished strip, with bad news in Section 3.19 and some possible good news in Section 3.20, suggesting that we promote the dual pairing $\big(\dot{S}(\mathbb{R}^n), \dot{S}'(\mathbb{R}^n)\big)$ (and spaces within) from troublesome offspring of $\big(S(\mathbb{R}^n), S'(\mathbb{R}^n)\big)$ to respected junior partner. However, the outcome remains to be seen. More details about motivation and intentions may be found in the Introduction.