

**IRMA Lectures in Mathematics and Theoretical Physics 25**

Edited by Christian Kassel and Vladimir G. Turaev

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Takashi Shioya

# Metric Measure Geometry

Gromov's Theory of  
Convergence and Concentration of  
Metrics and Measures



European Mathematical Society

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