Foreword

Mathematicians have ambiguous relations with the history of their discipline. They experience pride in describing how important new concepts emerged gradually or suddenly, but sometimes tend to prettify the history, carried away with imaginings of how ideas might have developed in harmonious and coherent fashion. This tendency has sometimes irritated professional historians of science, well aware that the development has often been much more tortuous.

It is our implicit belief that the uniformization theorem is one of the major results of 19th century mathematics. In modern terminology its formulation is simple:

Every simply connected Riemann surface is isomorphic to the complex plane, the open unit disc, or the Riemann sphere.

And one can even find proofs in the recent literature establishing it by means of not very complicated argumentation in just a few pages (see e.g. [Hub2006]). Yet it required a whole century before anyone managed to formulate the theorem and for a convincing proof to be given in 1907. The present book considers this maturation process from several angles.

But why is this theorem interesting? In the introduction to his celebrated 1900 article [Hil1900b] listing his 23 most significant open problems, David Hilbert proposed certain "criteria of quality" characterizing a good problem. The first of these requires that the problem be easy to state, and the uniformization theorem certainly satisfies this condition since its statement occupies only two lines! The second requirement — that the proof be beautiful — we leave to the reader to check. Finally, and perhaps most importantly, it should generate connections between different areas and lead to new developments. The reader will see how the uniformization theorem evolved in parallel with the emergence of modern algebraic geometry, the creation of complex analysis, the stirrings of functional analysis, the blossoming of the theory of linear differential equations, and the birth of topology. It is one of the guiding principles of 19th century mathematics. And furthermore Hilbert's twenty-second problem was directly concerned with uniformization.

We should give the reader fair warning that this book represents a rather modest contribution. Its authors are not historians — many of them can't even read German! They are mathematicians wishing to cast a stealthy glance at the past of this so fundamental theorem in the hope of bringing to light some of the beautiful — and potentially useful — ideas lying hidden in long-forgotten papers. Furthermore, the authors cannot claim to belong to the first rank of specialists in modern aspects of the uniformization theorem. Thus the present work is not a complete treatise on the subject, and we are aware of the gaps we should have plugged if only we had had the time.

Our exposition is perhaps somewhat unusual. We don't so much describe the history of a result as re-examine the old proofs with the eyes of modern mathematicians, querying their validity and attempting to complete them where they fail, first as far as possible within the context of the background knowledge of the period in question, or, if that turns out to be too difficult, then by means of modern mathematical tools not available at the time. Although the proofs we arrive at as a result are not necessarily more economical than modern ones, it seems to us that they are superior in terms of ease of comprehension. The reader should not be surprised to find many anachronisms in the text — for instance calling on Sobolev to rescue Riemann! Nor should he be surprised that results are often stated in a much weaker form than their modern-day versions — for example, we present the theorem on isothermal coordinates, established by Ahlfors and Bers under the general assumption of measurability, only in the analytic case dealt with by Gauss. Gauss' idea seems to us so limpid as to be well worth presenting in his original context.

We hope that this book will be of use to today's mathematicians wishing to glance back at the history of their subject. But we also believe that it can be used to provide masters-level students with an illuminating approach to concepts of great importance in contemporary research.

The book was conceived as follows: In 2007 fifteen mathematicians foregathered at a country house in *Saint-Germain-la-Forêt*, Sologne, to spend a week expounding to one another fifteen different episodes from the history of the uniformization theorem, given its first complete proof in 1907. It was thus a week commemorating a mathematical centenary! Back home, the fifteen edited their individual contributions, which were then amalgamated. A second retreat in the same rural setting one year later was devoted to intensive collective rewriting, from which there emerged a single work in manuscript form. After multiple further rewriting sessions, this time in small subsets of the fifteen, the present book ultimately materialized.

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