# Preface to the second edition

The first edition of this book was published about four years ago. My goal was to write an introductory graduate level textbook mostly on topological aspects of Noncommutative Geometry to fill a certain gap in the literature. When my publisher told me that the book is out of print and there is still good demand for it and we should perhaps think about a second edition, I felt vindicated!

Not much has changed in this second edition. I have just added two new sections, and deleted none. One is a very brief, and I am afraid awfully terse, introduction to a very recent development in the subject concerning the Gauss–Bonnet theorem and scalar curvature for curved noncommutative tori, and the second is a brief introduction to Hopf cyclic cohomology. A proper treatment of curvature in noncommutative geometry requires tools beyond the scope of this book and can only be adequately treated with much extra preparatory material. The bibliography is extended and some new examples are offered.

The progress in noncommutative geometry in the last thirty five years can be roughly divided into three phases in chronological order: *topological, spectral, and arithmetical.* A student of the subject is well advised to follow the historical development and acquaint herself with all three aspects before focusing on a particular research topic. As said before, this book mostly covers topological issues: noncommutative spaces, cyclic cohomology and its relation with $K$-theory and $K$-homology, noncommutative index theory, and noncommutative quotients. Much has happened in the field between these two editions which unfortunately cannot be dealt with here without substantially increasing the number of pages. For an introduction to the circle of ideas relating number theory and algebraic geometry to noncommutative geometry I refer to Connes and Consani [43] and their last paper [44] and references therein. For relations with spectral geometry, high energy physics, and number theory, the reader is referred to Connes–Marcolli's monograph [55].

**How to use this book:** Noncommutative geometry draws on many ideas and techniques from different areas of mathematics which are usually not all mastered by a graduate student. Given this, I would like to say a few words about using this book as a textbook for a graduate course. My experience is that it is not possible to cover all the material in one term and one really needs a full year course for that. One can teach a one term course based on Chapter 3, cyclic cohomology. Here the instructor needs to fill in background material on homological algebra that I have assumed and is not discussed in the text: resolutions, derived functors, long exact sequences, and spectral sequences. I have also assumed basic knowledge of algebraic topology based on differential forms and de Rham cohomology. Alternatively a one term course can be taught based on Chapter 4, Connes–Chern character. This chapter is more analysis based and assumes basic notions of functional analysis and operator theory. The instructor can add extra material on $K$-theory, $K$-homology, spectral triples, pseudodifferential operators, and the index theorem. I think basic ideas of Chapter 1 on duality theorems, and Chapter 2 on noncommutative quotients should be incorporated in any introductory course on the subject.

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