

# Contents

Preface to the second edition	vii
Introduction	xi
<b>1 Examples of algebra-geometry correspondences</b>	1
1.1 Locally compact spaces and commutative $C^*$ -algebras . . . . .	1
1.1.1 The spectrum . . . . .	2
1.1.2 The Gelfand transform . . . . .	3
1.1.3 Noncommutative spaces . . . . .	7
1.1.4 Noncommutative spaces from groups . . . . .	9
1.1.5 Noncommutative tori . . . . .	14
1.2 Vector bundles, finite projective modules, and idempotents . . . . .	18
1.3 Affine varieties and finitely generated commutative reduced algebras .	24
1.4 Affine schemes and commutative rings . . . . .	26
1.5 Compact Riemann surfaces and algebraic function fields . . . . .	28
1.6 Sets and Boolean algebras . . . . .	29
1.7 From groups to Hopf algebras and quantum groups . . . . .	30
1.7.1 Symmetry in noncommutative geometry . . . . .	39
<b>2 Noncommutative quotients</b>	47
2.1 Groupoids . . . . .	47
2.2 Groupoid algebras . . . . .	52
2.3 Morita equivalence . . . . .	64
2.4 Morita equivalence for $C^*$ -algebras . . . . .	73
2.5 Noncommutative quotients . . . . .	79
2.6 Sources of noncommutative spaces . . . . .	86
<b>3 Cyclic cohomology</b>	87
3.1 Hochschild cohomology . . . . .	89
3.2 Hochschild cohomology as a derived functor . . . . .	95
3.3 Deformation theory . . . . .	102
3.4 Topological algebras . . . . .	112
3.5 Examples: Hochschild (co)homology . . . . .	115
3.6 Cyclic cohomology . . . . .	124
3.7 Connes' long exact sequence . . . . .	136
3.8 Connes' spectral sequence . . . . .	140
3.9 Cyclic modules . . . . .	143
3.10 Examples: cyclic cohomology . . . . .	148

3.11 Hopf cyclic cohomology . . . . .	154
<b>4 Connes–Chern character</b>	163
4.1 Connes–Chern character in $K$ -theory . . . . .	163
4.1.1 Basic $K$ -theory . . . . .	164
4.1.2 Pairing with cyclic cohomology . . . . .	166
4.1.3 Noncommutative Chern–Weil theory . . . . .	172
4.1.4 The Gauss–Bonnet theorem and scalar curvature in noncommutative geometry . . . . .	176
4.2 Connes–Chern character in $K$ -homology . . . . .	177
4.3 Algebras stable under holomorphic functional calculus . . . . .	194
4.4 A final word: basic noncommutative geometry in a nutshell . . . . .	199
<b>Appendices</b>	201
<b>A Gelfand–Naimark theorems</b>	201
A.1 Gelfand’s theory of commutative Banach algebras . . . . .	201
A.2 States and the GNS construction . . . . .	205
<b>B Compact operators, Fredholm operators, and abstract index theory</b>	212
<b>C Projective modules</b>	219
<b>D Equivalence of categories</b>	221
Bibliography	223
Index	235