Introduction

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MATHEON, a word invented by students of the Universität der Künste Berlin, has by now become an international trademark. It is the name of a mathematical research center in Berlin and stands for creative application-driven research in mathematics aiming at an interdisciplinary and cooperative solution of challenging problems in key technologies.

The DFG Research Center MATHEON has changed the mathematical landscape during the last 12 years – not only in the Berlin region but far beyond its geographical and disciplinary boundaries. The financial support of MATHEON by the German Science Foundation (DFG) terminates, as planned, after 12 years of significant funding at the end of May 2014; but its spirit will continue in Berlin and elsewhere in various new forms of coordinated research programs. Most importantly, MATHEON will survive as an organization as a part of the newly created Einstein Center Mathematics (briefly ECMath) in Berlin.

The goals of MATHEON were described concisely in the Executive Summary of the proposal for its establishment in 2001 as follows:

Key technologies become more complex, innovation cycles get shorter. Flexible mathematical models open new possibilities to master complexity, to react quickly, and to explore new smart options. Such models can only be obtained via abstraction. This line of thought provides our global vision: Innovation needs flexibility, flexibility needs abstraction, the language of abstraction is mathematics. But mathematics is not only a language, it adds value: theoretical insight, efficient algorithms, optimal solutions. Thus, key technologies and mathematics interact in a joint innovation process.

The mission of the Center is to give a strong push to the role of mathematics in this interactive process. The Center's research program is application-driven. Its implementation will have a strong impact on the development of mathematics itself and will define a new stage of interand transdisciplinary cooperation.

This book proves that these promises were kept and that solutions to most of the detailed goals, described in the application booklet, were delivered. More than that, the way MATHEON was organized, how it fostered cooperation and competition, how it decided which projects to support, which not and which to terminate, how it was able to bring together highly creative mathematicians from a wide range of mathematical subfields and institutions to work jointly on big industrial challenges, has influenced the creation of mathematical research centers the world over. MATHEON representatives were not only supportive for other mathematical institutions, they also helped to create clusters and centers in other scientific disciplines in many ways.

The most important "MATHEON output", though, was the education of a new generation of mathematicians who ignore the artificial boundaries of pure and applied mathematics and who have understood that mathematics is not only one of the greatest intellectual achievements of mankind, but also an extremely powerful and versatile tool that can contribute to almost every area of science, technology, economics, and society. More than 90 offers for professorships were made to young researchers who went through the "MATHEON school" in the last 12 years. This new generation of professors is spreading the "MATHEON ideals"; and an even larger number of former students, who were employed in MATHEON projects and are now working in industry or elsewhere, are carrying the MATHEON approach to the solution of real-world problems to the world outside of academia, making the road to the employment of mathematics in industry much shorter and faster.

We believe that the challenges for the future of mankind (ecology, energy, use of scarce resources, transport, communication, production, new materials, health, etc.) cannot be solved without mathematical modelling, simulation, control, and optimization. Integrated and interdisciplinary approaches are necessary. Mathematics is an indispensable part of these activities. The work that has been done in MATHEON from basic theory, via algorithm design and coding, to implementation in practice, is a small but important step in this direction.

Let us briefly describe how MATHEON came into being.

In the year 2000, frequencies for the new mobile telephone standard UMTS were auctioned out in Germany. The German Government decided to invest a small part (but in monetary terms still a large amount) of the gigantic gains of about 50 billion Euros into the support of research. The DFG had the idea to create a new coordinated program, entitled "DFG-Forschungszentren", DFG Research Centers. After a first call without any particular specification, DFG issued a call for the establishment of one DFG Research Center in "Modellierung und Simulation in den Ingenieur-, Natur- und Sozialwissenschaften" (modelling and simulation in engineering, natural and social sciences). A group of Berlin mathematicians from the mathematical institutes of the three big universities in Berlin (Freie Universität (FU), Humboldt-Universität (HU) and Technische Universität (TU)) and from the two Berlin-based mathematical research institutes (Weierstraß-Institut für Angewandte Analysis und Stochastik im Forschungsverbund Berlin e.V. (WIAS) and Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)) applied for this center and won the scientific competition. The final decision, made by the DFG-Hauptausschuss on May 8, 2002, came only after some delay, since the organizational set-up as a joint center shared by 3 universities and 2 research institutes was questioned. This set-up was, however, some years later used as the role model for the design of the next coordinated program, called "clusters" in the so-called Excellence Initiative of the German Federal Ministry of Education and Research (BMBF) and DFG.

The conceptual design of a DFG Research Center in application-driven mathematics began in the fall of the year 2000, an application of about 400 pages for the creation of a DFG Research Center, to be entitled "Mathematics for key technologies: Modelling, simulation, and optimization of real-world processes" (and named FZT 86 by DFG), was submitted in November 2001. On June 1, 2002, immediately after the positive decision, FZT 86 took up its operation. In a competition for an appropriate name, better than FZT 86 and much shorter than the full title, students of the Universität der Künste created the word MATHEON, which everyone likes and which has significantly contributed to the visibility of the center.

Another important discussion that took place in the start-up phase was the internal organization of the center. To underline the application-driven character of mathematical modelling,

simulation, and optimization, it was decided to organize the structure of mathematical contributions within seven *Application Areas*

- A Life Sciences,
- B Networks,
- C Production,
- D Electronic and Photonic Devices,
- E Finance,
- F Visualization, and
- ZE *Education.*

A detailed description of the work and a summary of the highlights in these application areas is the main topic of this book. Mathematical research in key technologies, however, can typically not be restricted to one mathematical discipline; it requires a coordinated effort of different mathematical fields. Mathematics plays a transversal role in the solution of application problems. For this reason we complemented the application areas in a matrix structure with the three *Mathematical Fields*

- I Optimization and Discrete Mathematics,
- II Numerical Analysis and Scientific Computing, and
- III Applied and Stochastic Analysis.

With more than 45 professors involved, MATHEON includes a critical mass of mathematicians, excelling in all these fields, who have been and are still willing to take up the challenge of doing application-driven basic research to advance the key technologies in focus.

An important aspect of the MATHEON organization and a major factor for its scientific success and worldwide visibility is the strong quality control that has been executed by MATH-EON's governing bodies. Every research project has to present its research plans as well as the achievements at the annual *center days* at which all projects are evaluated according to a variety of criteria. Besides the usual scientific criteria for the evaluation of basic research, such as publications in major scientific journals, invitations to plenary lectures at international conferences, or scientific distinctions, also interdisciplinary cooperation and transfer into industry or other sciences, as well as outreach to the general public, are addressed in the evaluation process. This philosophy (which required a change of mentality for some of the participating scientists) has led to a large number of joint projects of MATHEON scientists within further interdisciplinary cooperations, such as the current participation in eight DFG "Sonderforschungsbereiche" (Collaborative Research Centers), eight DFG Priority Programs, five DFG Research Training Groups as well as numerous other interdisciplinary research activities. MATHEON is strongly involved in PVComB - Competence Center Thin-Film- and Nanotechnology for Photovoltaics Berlin, an innovation cluster funded by the federal ministry BMBF. At the same time the cooperation with industry (viewed in a broad sense comprising also services, banks or hospitals) has led to major impacts including the formation of a large number of spin-off companies, which all have survived in the market, and the foundation of the BMBF Research Campus MODAL.

From its beginning, MATHEON has put the education of the next generation of researchers into its focus. It is our understanding that the foundations for the interest in science and technology have to be laid in school as early as possible and have to be supported during the whole education chain from kindergarten up to PhD. To get actively involved in this process, MATHEON has initiated the *mathematischer Adventskalender* (mathematics Advent calendar), where

every day in December until Christmas Eve high-school students of the last three school years are challenged to solve a mathematical question derived from a real MATHEON application. This calendar is now also offered, in cooperation with the German Mathematical Society, in three versions aiming at different school grades. In 2013 this competition had the astonishing number of exactly 164 372 participants; and although all problems are stated in German only, students from 63 different countries took part.

Another successful activity created by MATHEON is a new type of sports event, called MATHEATHION, where kids have to run on a track and have to solve mathematics questions at certain intervals on the way. This event came into being within the program of the IAAF World Championship in Athletics in 2009 in Berlin. Today MATHEATHION competitions are held in schools all over Germany.

Recognizing the large demand for more education in the STEM fields (science, technology, engineering and mathematics), MATHEON has started to cooperate with several mathematics oriented high schools, where mathematically gifted students get special support and where credits of the undergraduate curriculum at the university can already be gained. With the support of MATHEON researchers, teams from these schools have been extremely successful in (Inter-)National Students' Competitions such as "ZeroRobotics/SPHERES" with MIT, ESA and NASA.

MATHEON also created special mathematics labs for high-school students and many other activities which are described in more detail in the section on education and outreach. Special outreach highlights were the MATHEON event for the celebration of the German award "Land of Ideas", the opening of the Advent calendar on November 30, 2007 and the celebration of 10th anniversary of MATHEON in 2012, which drew huge crowds of more than 1000 participants each.

MATHEON had also been instrumental in 2006 in the creation of the Berlin Mathematical School (BMS), a joint internationally oriented graduate school of the three big Berlin universities which is supported within the German Excellence Initiative, and in 2011 in the foundation of the German Center for Teacher Education in Mathematics, which is funded by the Telekom Foundation.

Within MATHEON it was possible to employ a large number of new professors. There was support for six new tenured full professors for each four-year funding period, and MATHEON could hire seven young scientists as junior research group leaders on temporary (six year) positions on the assistant professor level. The latter program was very successful. All the heads of these groups were very quickly recruited by other institutions to permanent positions so that the hiring process was in a continuous flux. At no point in time all seven positions where filled simultaneously.

On average, at any point in time over the last 12 years, more than fifty DFG funded mathematicians have been working in the MATHEON research projects and about additional 150 persons, funded by FU, TU, HU, WIAS and ZIB and a variety of other sources, have been working on MATHEON related projects. These activities brought about on the one hand a large number of major scientific contributions, excellently educated mathematics students and a new generation of professors, and on the other hand the solution of many relevant industrial problems, the foundation of successful spin-off companies and a fresh look at mathematics. MATHEON has shown convincingly that mathematics is not only a demanding intellectual endeavor, but also an important production factor. The high visibility and agility of the MATHEON research was also internationally recognized by a large number of Leibniz prizes, ICIAM prizes, SIAM

fellowships, ERC grants, dissertation prizes, as well as the election of MATHEON representatives into governing or advisory boards of national and international scientific organizations.

This book presents in its seven chapters the research highlights of the work in the MATHEON application areas. The introductions to these chapters summarize the contributions. The book also includes a large number of separate showcases illustrating some of the success stories in a way that can be easily understood by the general public.

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