Preface

Andrzej Bobola Maria Schinzel, born on April 5, 1937 at Sandomierz (Poland), is well known for his original results in various areas of number theory appearing in over 200 research papers, of which the first thirty were published while he was an undergraduate at the Warsaw University. Working under the guidance of Wacław Sierpiński, he became interested in elementary number theory, and the subjects of his early papers range from properties of arithmetical functions, like Euler's φ-function or the number of divisors, to Diophantine equations. Paul Erdős, a big champion of elementary number theory, wrote in his letter to Sierpiński of October 23, 1960—when Andrzej was studying at the University of Cambridge under the supervision of Harold Davenport—*"Schinzel's completion of my proof is much simpler than anything I had in mind"*. Many mathematicians cooperating with Andrzej Schinzel could repeat the words of Erdős.

Since completing his studies at Warsaw University in 1958, Andrzej Schinzel has been employed by the Institute of Mathematics of the Polish Academy of Sciences, where he obtained his Ph.D. in 1960. On his return from a Rockefeller Foundation Fellowship at the University of Cambridge and the University of Uppsala (where he studied under Trygve Nagell) he completed his *habilitation* in 1962. In 1967 he was promoted to Associate Professor, and in 1974 to Full Professor. In 1979 he was elected to Corresponding Member of the Polish Academy of Sciences and in 1994 to Full Member.

Andrzej Schinzel's very first paper (1)—published at the age of 17—is a postscript to a result of H.-E. Richert who proved a general theorem about partitions of integers into distinct summands from a given set which implied in particular that every integer > 33 is a sum of distinct triangular numbers. Schinzel observed that every integer > 51 is a sum of at most four distinct triangular numbers. The favorite subject of the early research of Schinzel, Euler's totient function, is considered here in five papers; the earliest one, published in 1954, is not included (2). In 1958 a joint work **J1** with Sierpiński appeared, analyzing various consequences of the conjecture stating that if $f_1, \ldots, f_s \in \mathbb{Z}[x]$ are irreducible polynomials having positive leading coefficients and there is no natural number > 1 that is a divisor of each of the numbers $f_1(n) \cdots f_s(n)$ for n being an integer then for infinitely many natural n the values $f_1(n), \ldots, f_s(n)$ are primes. This celebrated conjecture—with many unexpected consequences—is called *"Schinzel's Hypothesis H"*.

Schinzel's doctoral thesis **B1** dealt with the period of a class of continued fractions and was related (see **B2**) to a question concerning pseudo-elliptic integrals, considered already

---

(1) *Sur la décomposition des nombres naturels en sommes de nombres triangulaires distincts*, Bull. Acad. Polon. Sci. Cl. III 2 (1954), 409–410.

(2) *Sur quelques propriétés des fonctions $\varphi(n)$ et $\sigma(n)$*, Bull. Acad. Polon. Sci. Cl. III 2 (1954), 463–466 (with W. Sierpiński).

by N. H. Abel in the very first volume of Crelle's Journal. In his habilitation thesis—consisting of four papers **I1**, **I2**, **I3** and a paper not included ([3])—Schinzel generalized a classical theorem of Zsigmondy of 1892 (often called the Birkhoff–Vandiver theorem) on primitive divisors.

The central theme of Schinzel's work is arithmetical and algebraic properties of polynomials in one or several variables, in particular questions of irreducibility and zeros of polynomials. To this topic he devoted about one-third of his papers and two books ([4]). In the books Schinzel presented several classical results and included many extensions, improvements and generalizations of his own.

Undoubtedly Schinzel and his beloved journal *Acta Arithmetica* influenced many mathematicians, stimulating their thinking and mathematical careers. Andrzej Schinzel has since 1969 been the editor of this first international journal devoted exclusively to number theory, being a successor of his teacher W. Sierpiński. Among the other editors of Acta Arithmetica during these years were/are J. W. S. Cassels, H. Davenport, P. Erdős, V. Jarník, J. Kaczorowski, Yu. V. Linnik, L. J. Mordell, W. M. Schmidt, V. G. Sprindzhuk, R. Tijdeman and P. Turán. These people and the other outstanding mathematicians from the advisory board of Acta Arithmetica have determined the line of the journal.

Andrzej Schinzel's work has been influential in the development of many areas of mathematics, and his 70th birthday gives us an opportunity to honor his accomplishments by putting together his most important papers. This selection of Schinzel's papers—published during more than five decades—is divided into two volumes containing 100 articles. We have asked some outstanding mathematicians for commentaries to the selected papers. Also included is a list of unsolved problems and unproved conjectures proposed by Schinzel in the years 1956–2006, arranged chronologically. The first volume covers six themes:

A. Diophantine equations and integral forms (with commentaries by Robert Tijdeman)
B. Continued fractions (with commentaries by Eugène Dubois)
C. Algebraic number theory (with commentaries by David W. Boyd and Donald J. Lewis)
D. Polynomials in one variable (with commentaries by Michael Filaseta)
E. Polynomials in several variables (with commentaries by Umberto Zannier)
F. Hilbert's Irreducibility Theorem (with commentaries by Umberto Zannier)

The second volume contains papers covering seven themes:

G. Arithmetic functions (with commentaries by Kevin Ford)
H. Divisibility and congruences (with commentaries by Hendrik W. Lenstra, Jr.)
I. Primitive divisors (with commentaries by Cameron L. Stewart)
J. Prime numbers (with commentaries by Jerzy Kaczorowski)
K. Analytic number theory (with commentaries by Jerzy Kaczorowski)
L. Geometry of numbers (with commentaries by Wolfgang M. Schmidt)
M. Other papers (with commentaries by Stanisław Kwapień and Endre Szemerédi)

([3]) *The intrinsic divisors of Lehmer numbers in the case of negative discriminant*, Ark. Mat. 4 (1962), 413–416.

([4]) *Selected Topics on Polynomials*, XXII+250 pp., University of Michigan Press, Ann Arbor 1982, and *Polynomials with Special Regard to Reducibility*, X+558 pp., Encyclopaedia of Mathematics and its Applications 77, Cambridge Univ. Press, Cambridge 2000.

Many people helped with the editing of the volumes. First of all, we gratefully thank the authors of the commentaries we had the pleasure to work with. Second, our special thanks go to Stanisław Janeczko, Director of the Institute of Mathematics, Polish Academy of Sciences, for his support, and to Manfred Karbe, Publishing Director of the European Mathematical Society Publishing House, for his invaluable assistance during the work on the Selecta. Third, we wish to thank Jerzy Browkin for reading the papers and some corrections, and Jan K. Kowalski for retyping the papers and offering valuable suggestions for improving the presentation of the material. Finally, we wish to express our gratitude to the staff of the European Mathematical Society Publishing House, especially to Irene Zimmermann, for the very pleasant cooperation.

We have decided to unify some notations, using the "blackboard bold" type for most common sets; so $\mathbb{C}$, $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Q}_p$ and $\mathbb{F}_p$ always stand for the complex, real, rational, p-adic and finite fields respectively; $\mathbb{Z}$, $\mathbb{Z}_p$ for the rings of integers and p-adic integers; and $\mathbb{N}$, $\mathbb{N}_0$ for the sets of positive and nonnegative integers. The greatest common divisor of integers $a_1, a_2, \dots, a_n$ is denoted by $(a_1, a_2, \dots, a_n)$. If $(a_1, a_2, \dots, a_n) = 1$, the integers are called relatively prime, and if $(a_i, a_j) = 1$ for any $1 \leqslant i \neq j \leqslant n$, the integers are called coprime. As usual, for $x \in \mathbb{R}$ set $[x] = \max\{n \in \mathbb{Z} : n \leqslant x\}$, $\lceil x \rceil = \min\{n \in \mathbb{Z} : x \leqslant n\}$ and $\|x\| = \min\{x - [x], \lceil x \rceil - x\}$. We denote by $\{x\}$ the fractional part of x. Lines where minor corrections of the original text have been made are marked "c" in the left margin.

March 2007

Henryk Iwaniec
Władysław Narkiewicz
Jerzy Urbanowicz