

Contents

1	Introduction	1
1.1	History of the problem	1
1.2	Conjectures, scope of the book	3
2	High frequency delocalisation	15
2.1	L^p -norms as measures of delocalisation?	16
2.2	The Shnirelman theorem	18
2.3	Overview of pseudo-differential operators	20
2.4	The geodesic flow and the ergodicity assumption	32
2.5	Proof of the Quantum Ergodicity theorem	34
3	Entropy and support of semiclassical measures	37
3.1	Quantum Unique Ergodicity conjecture	37
3.2	Statement of results	38
3.3	What is entropy?	41
3.4	Proof of the entropy result (Theorem 3.1)	45
3.5	A few words on Dyatlov and Jin's result	55
4	Quantum ergodicity on regular graphs	57
4.1	Regular graphs: quantum ergodicity	58
4.2	Harmonic analysis on the $(q + 1)$ -regular tree	66
4.3	Proof of Theorem 4.2	71
5	Quantum ergodicity on the sphere	81
6	Quantum ergodicity on non-regular graphs	85
6.1	Introduction	85
6.2	A proof based on the non-backtracking random walk	89
6.3	Adaptation to non-regular graphs	99
6.4	Trees of finite cone type and their stochastic perturbations	107
6.5	The measure $\langle a \rangle_\gamma$: one example	115
7	Backhausz and Szegedy's theorem	117
7.1	Gaussianity of eigenfunctions	117
7.2	Elements of the proof	120
	References	127