

Preface

One of the most basic and important questions in PDE is that of regularity. It is also a unifying problem in the field, since it affects all kinds of PDEs. A classical example is Hilbert's XIXth problem (1900), which roughly speaking asked to determine whether all solutions to uniformly elliptic variational PDEs are smooth. The question was answered positively by De Giorgi and Nash in 1956 and 1957, and it is now one of the most famous and important theorems in the whole field of PDE.

The question of regularity has been a central line of research in elliptic PDE since the mid-20th century, with extremely important contributions by Nirenberg, Caffarelli, Krylov, Evans, Figalli, and many others. Their works have enormously influenced many areas of mathematics linked one way or another with PDE, including: harmonic analysis, calculus of variations, differential geometry, geometric measure theory, continuum and fluid mechanics, probability theory, mathematical physics, and computational and applied mathematics.

This text emerged from two PhD courses on elliptic PDE given by the second author at the University of Zürich in 2017 and 2019. It aims to provide a self-contained introduction to the regularity theory for elliptic PDE, focusing on the main ideas rather than proving all results in their greatest generality. The book can be seen as a bridge between an elementary PDE course and more advanced textbooks such as [40] or [16]. Moreover, we believe that the present selection of results and techniques complements nicely other books on elliptic PDE such as [30], [41], and [51], as well as the recent book [3]. For example, we give an alternative proof of the Schauder estimates (due to L. Simon) which is not contained in other textbooks; we prove some basic results for fully nonlinear equations that are not covered in [16]; and we also include a detailed study of the obstacle problem, often left to more specialized textbooks such as [39] or [71]. Furthermore, at the end of Chapters 3, 4, and 5 we provide a review of some recent results and open problems.

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