

Contents

1	Introduction	1
1.1	Background	1
1.2	The main problem	2
1.3	Master functions and master differential equations	3
1.4	Borel–Laplace multitransforms	5
1.5	Main results	6
1.6	Dubrovin conjecture for Hirzebruch surfaces	8
1.7	Plan of the paper	11
2	Cyclic stratum of Frobenius manifolds	13
2.1	Frobenius manifolds	13
2.2	Semisimple points and bifurcation set	15
2.3	Extended deformed connection	16
2.4	Cyclic stratum, and cyclic (co)frame	17
2.5	Properties of the function $\det \Lambda$	18
2.6	Geometry of the complement of the cyclic stratum in $\mathbb{P}^1 \times M$	21
2.7	Master differential equation and master functions	24
3	Gromov–Witten theory	27
3.1	Notations and conventions	27
3.2	Descendant Gromov–Witten invariants	28
3.3	Quantum cohomology	29
4	Monodromy data of quantum cohomology	31
4.1	Topological-enumerative solution	31
4.2	Stokes rays and ℓ -chamber decomposition	31
4.3	Stokes fundamental solutions at $z = \infty$	32
4.4	Monodromy data	33
4.5	Natural transformations of monodromy data	34
4.6	Action of the braid group \mathcal{B}_n	35
5	J-function and quantum Lefschetz theorem	39
5.1	J -function and master functions	39
5.2	Twisted Gromov–Witten invariants	40
5.3	Quantum Lefschetz theorem	41
5.4	Inequality for dimensions of spaces of master functions	42

6	Borel–Laplace (α, β)-multitransforms	45
6.1	Algebras of Ribenboim’s generalized power series	45
6.2	The algebra $\mathcal{F}_\kappa(A)$	46
6.3	Formal Borel–Laplace (α, β) -multitransforms	47
6.4	Analytic Borel–Laplace (α, β) -multitransforms	48
6.5	Analytification of elements of $\mathcal{F}_\kappa(A)$	49
7	Integral representations of solutions of qDEs	53
7.1	J_X -function as element of $\mathcal{F}_\kappa(X)$	53
7.2	Integral representations of the first kind	54
7.3	Integral representations of the second kind	56
7.4	Master functions as Mellin–Barnes integrals	58
8	Dubrovin conjecture	61
8.1	Exceptional collections and exceptional bases	61
8.2	Mutations and helices	61
8.3	Γ -classes and graded Chern character	62
8.4	Statement of the conjecture	63
9	Quantum cohomology of Hirzebruch surfaces	67
9.1	Preliminaries on Hirzebruch surfaces	67
9.2	Classical cohomology of Hirzebruch surfaces	67
9.3	Quantum cohomology of Hirzebruch surfaces	69
10	Dubrovin conjecture for Hirzebruch surfaces \mathbb{F}_{2k}	73
10.1	\mathcal{A}_Λ -stratum and Maxwell stratum of $QH^\bullet(\mathbb{F}_{2k})$	73
10.2	Small qDE of \mathbb{F}_{2k}	74
10.3	Proof for $QH^\bullet(\mathbb{F}_{2k})$	75
11	Dubrovin conjecture for Hirzebruch surfaces \mathbb{F}_{2k+1}	81
11.1	\mathcal{A}_Λ -stratum and Maxwell stratum of $QH^\bullet(\mathbb{F}_{2k+1})$	81
11.2	Small qDE of \mathbb{F}_1	81
11.3	Coordinates on $\mathcal{S}(\mathbb{P}^1) \otimes \mathcal{S}(\mathbb{P}^2)$	84
11.4	Solutions of qDE of \mathbb{F}_1 as Laplace $(1, 2; \frac{1}{2}, \frac{1}{3})$ -multitransforms	85
11.5	Basis of solutions Υ of $\mathcal{S}(\mathbb{F}_1)$	92
11.6	Asymptotics of Laplace $(1, 2; \frac{1}{2}, \frac{1}{3})$ -multitransforms	93
11.7	Stokes basis of the qDE of \mathbb{F}_1	99
11.8	Computation of the central connection and Stokes matrices	102

A Proof of Theorem 5.1.2 107

B Coefficients $\mathcal{A}_j^{(i)}$ and $\mathcal{B}_j^{(i)}$ 111

References 119