

## Appendix B

### Coefficients $\mathcal{A}_j^{(i)}$ and $\mathcal{B}_j^{(i)}$

The coefficients  $\mathcal{A}_j^{(i)}$  and  $\mathcal{B}_j^{(i)}$ , introduced in equations (11.4.4) and (11.4.5), are

$$\mathcal{A}_1^{(1)}(m, n) = -\frac{8}{9}mn^2 A_{0,0} B_{0,0},$$

$$\mathcal{A}_2^{(1)}(m, n) = \frac{8}{9}n^2 A_{0,0} B_{0,0},$$

$$\mathcal{A}_3^{(1)}(m, n) = \frac{8}{9}m A_{0,0} B_{0,0},$$

$$\begin{aligned} \mathcal{A}_1^{(2)}(m, n) = & \frac{4}{9}n(4mn A_{0,0} B_{0,0} H_m + 6mn A_{0,0} B_{0,0} H_n - 4mn A_{0,1} B_{0,0} \\ & - 3mn A_{0,0} B_{0,1} - 4mn A_{0,0} B_{0,0} \psi^{(0)}(m+n+1) \\ & - 4m A_{0,0} B_{0,0} - 2n A_{0,0} B_{0,0}), \end{aligned}$$

$$\begin{aligned} \mathcal{A}_2^{(2)}(m, n) = & -\frac{4}{9}n(4n A_{0,0} B_{0,0} H_m + 6n A_{0,0} B_{0,0} H_n \\ & - 4n A_{0,0} B_{0,0} \psi^{(0)}(m+n+1) - 4n A_{0,1} B_{0,0} \\ & - 3n A_{0,0} B_{0,1} - 4 A_{0,0} B_{0,0}), \end{aligned}$$

$$\begin{aligned} \mathcal{A}_3^{(2)}(m, n) = & -\frac{4}{9}(6m A_{0,0} B_{0,0} H_n + 4m A_{0,0} B_{0,0} H_m \\ & - 4m A_{0,0} B_{0,0} \psi^{(0)}(m+n+1) - 4m A_{0,1} B_{0,0} \\ & - 3m A_{0,0} B_{0,1} - 2 A_{0,0} B_{0,0}), \end{aligned}$$

$$\begin{aligned} \mathcal{A}_1^{(3)}(m, n) = & -\frac{2}{9}(24mn^2 A_{0,0} B_{0,0} H_m H_n - 24mn^2 A_{0,1} B_{0,0} H_n \\ & - 12mn^2 A_{0,0} B_{0,1} H_m - 8mn^2 A_{0,0} B_{0,0} H_m \psi^{(0)}(m+n+1) \\ & - 18mn^2 A_{0,0} B_{0,0} H_n \psi^{(0)}(m+n+1) - 16mn A_{0,0} B_{0,0} H_m \\ & - 12mn A_{0,0} B_{0,0} H_n - 12n^2 A_{0,0} B_{0,0} H_n + 12mn^2 A_{0,1} B_{0,1} \\ & + 9mn^2 A_{0,0} B_{n,2} + 5mn^2 A_{0,0} B_{0,0} \psi^{(0)}(m+n+1)^2 \\ & + 4n^2 A_{0,0} B_{0,0} \psi^{(0)}(m+n+1) \\ & + 5mn^2 A_{0,0} B_{0,0} \psi^{(1)}(m+n+1) \\ & + 8mn^2 A_{0,1} B_{0,0} \psi^{(0)}(m+n+1) \\ & + 9mn^2 A_{0,0} B_{0,1} \psi^{(0)}(m+n+1) \\ & + 16mn A_{0,1} B_{0,0} + 6mn A_{0,0} B_{0,1} \\ & + 12mn A_{0,0} B_{0,0} \psi^{(0)}(m+n+1) \\ & + 2m A_{0,0} B_{0,0} + 6n^2 A_{0,0} B_{0,1} + 8n A_{0,0} B_{0,0}), \end{aligned}$$

$$\begin{aligned}
\mathcal{A}_2^{(3)}(m, n) = & \frac{2}{9} (24n^2 A_{0,0} B_{0,0} H_m H_n - 12n^2 A_{0,0} B_{0,1} H_m \\
& - 8n^2 A_{0,0} B_{0,0} H_m \psi^{(0)}(m+n+1) \\
& - 18n^2 A_{0,0} B_{0,0} H_n \psi^{(0)}(m+n+1) \\
& - 16n A_{0,0} B_{0,0} H_m - 24n^2 A_{0,1} B_{0,0} H_n - 12n A_{0,0} B_{0,0} H_n \\
& + 5n^2 A_{0,0} B_{0,0} \psi^{(0)}(m+n+1)^2 \\
& + 5n^2 A_{0,0} B_{0,0} \psi^{(1)}(m+n+1) \\
& + 8n^2 A_{0,1} B_{0,0} \psi^{(0)}(m+n+1) \\
& + 9n^2 A_{0,0} B_{0,1} \psi^{(0)}(m+n+1) \\
& + 12n A_{0,0} B_{0,0} \psi^{(0)}(m+n+1) \\
& + 12n^2 A_{0,1} B_{0,1} + 9n^2 A_{0,0} B_{n,2} \\
& + 16n A_{0,1} B_{0,0} + 6n A_{0,0} B_{0,1} + 2A_{0,0} B_{0,0}),
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_3^{(3)}(m, n) = & \frac{2}{9} (24m A_{0,0} B_{0,0} H_m H_n - 24m A_{0,1} B_{0,0} H_n \\
& - 8m A_{0,0} B_{0,0} H_m \psi^{(0)}(m+n+1) \\
& - 18m A_{0,0} B_{0,0} H_n \psi^{(0)}(m+n+1) \\
& - 12m A_{0,0} B_{0,1} H_m - 12A_{0,0} B_{0,0} H_n + 9m A_{0,0} B_{n,2} \\
& + 5m A_{0,0} B_{0,0} \psi^{(0)}(m+n+1)^2 + 4A_{0,0} B_{0,0} \psi^{(0)}(m+n+1) \\
& + 8m A_{0,1} B_{0,0} \psi^{(0)}(m+n+1) + 9m A_{0,0} B_{0,1} \psi^{(0)}(m+n+1) \\
& + 5m A_{0,0} B_{0,0} \psi^{(1)}(m+n+1) + 12m A_{0,1} B_{0,1} + 6A_{0,0} B_{0,1}),
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_1^{(4)}(m, n) = & -\frac{2}{9} (-18mn^2 A_{0,0} H_m B_{n,2} - 2mn^2 A_{0,0} B_{0,0} H_m \psi^{(0)}(m+n+1)^2 \\
& - 6mn^2 A_{0,0} B_{0,0} H_n \psi^{(0)}(m+n+1)^2 \\
& - 6n^2 A_{0,0} B_{0,0} H_n \psi^{(0)}(m+n+1) \\
& + 12mn^2 A_{0,0} B_{0,0} H_m H_n \psi^{(0)}(m+n+1) \\
& - 12mn^2 A_{0,1} B_{0,0} H_n \psi^{(0)}(m+n+1) \\
& - 6mn^2 A_{0,0} B_{0,1} H_m \psi^{(0)}(m+n+1) \\
& - 2mn^2 A_{0,0} B_{0,0} H_m \psi^{(1)}(m+n+1) \\
& - 6mn^2 A_{0,0} B_{0,0} H_n \psi^{(1)}(m+n+1) \\
& + 24mn A_{0,0} B_{0,0} H_m H_n - 24mn A_{0,1} B_{0,0} H_n \\
& - 12mn A_{0,0} B_{0,1} H_m - 8mn A_{0,0} B_{0,0} H_m \psi^{(0)}(m+n+1) \\
& - 12mn A_{0,0} B_{0,0} H_n \psi^{(0)}(m+n+1) - 4m A_{0,0} B_{0,0} H_m \\
& - 12n A_{0,0} B_{0,0} H_n + 18mn^2 A_{0,1} B_{n,2} \\
& + mn^2 A_{0,0} B_{0,0} \psi^{(0)}(m+n+1)^3)
\end{aligned}$$

$$\begin{aligned}
& + n^2 A_{0,0} B_{0,0} \psi^{(0)}(m+n+1)^2 \\
& + 2mn^2 A_{0,1} B_{0,0} \psi^{(0)}(m+n+1)^2 \\
& + 3mn^2 A_{0,0} B_{0,1} \psi^{(0)}(m+n+1)^2 \\
& + 3mn^2 A_{0,0} B_{0,0} \psi^{(1)}(m+n+1) \psi^{(0)}(m+n+1) \\
& + 3n^2 A_{0,0} B_{0,1} \psi^{(0)}(m+n+1) \\
& + 6mn^2 A_{0,1} B_{0,1} \psi^{(0)}(m+n+1) \\
& + 9mn^2 A_{0,0} B_{n,2} \psi^{(0)}(m+n+1) \\
& + n^2 A_{0,0} B_{0,0} \psi^{(1)}(m+n+1) \\
& + mn^2 A_{0,0} B_{0,0} \psi^{(2)}(m+n+1) \\
& + 2mn^2 A_{0,1} B_{0,0} \psi^{(1)}(m+n+1) \\
& + 3mn^2 A_{0,0} B_{0,1} \psi^{(1)}(m+n+1) + 12mn A_{0,1} B_{0,1} \\
& + 4mn A_{0,0} B_{0,0} \psi^{(0)}(m+n+1)^2 \\
& + 2m A_{0,0} B_{0,0} \psi^{(0)}(m+n+1) \\
& + 4n A_{0,0} B_{0,0} \psi^{(0)}(m+n+1) \\
& + 8mn A_{0,1} B_{0,0} \psi^{(0)}(m+n+1) \\
& + 6mn A_{0,0} B_{0,1} \psi^{(0)}(m+n+1) \\
& + 4mn A_{0,0} B_{0,0} \psi^{(1)}(m+n+1) \\
& + 4m A_{0,1} B_{0,0} + 9n^2 A_{0,0} B_{n,2} + 6n A_{0,0} B_{0,1} + 2A_{0,0} B_{0,0}), \\
\mathcal{A}_2^{(4)}(m, n) = & \frac{2}{9} (-18n^2 A_{0,0} H_m B_{n,2} - 2n^2 A_{0,0} B_{0,0} H_m \psi^{(0)}(m+n+1)^2 \\
& - 6n^2 A_{0,0} B_{0,0} H_n \psi^{(0)}(m+n+1)^2 \\
& + 12n^2 A_{0,0} B_{0,0} H_m H_n \psi^{(0)}(m+n+1) \\
& - 12n^2 A_{0,1} B_{0,0} H_n \psi^{(0)}(m+n+1) \\
& - 6n^2 A_{0,0} B_{0,1} H_m \psi^{(0)}(m+n+1) \\
& - 2n^2 A_{0,0} B_{0,0} H_m \psi^{(1)}(m+n+1) \\
& - 6n^2 A_{0,0} B_{0,0} H_n \psi^{(1)}(m+n+1) \\
& + 24n A_{0,0} B_{0,0} H_m H_n - 12n A_{0,0} B_{0,1} H_m \\
& - 8n A_{0,0} B_{0,0} H_m \psi^{(0)}(m+n+1) \\
& - 12n A_{0,0} B_{0,0} H_n \psi^{(0)}(m+n+1) \\
& - 4A_{0,0} B_{0,0} H_m - 24n A_{0,1} B_{0,0} H_n \\
& + n^2 A_{0,0} B_{0,0} \psi^{(0)}(m+n+1)^3 \\
& + 2n^2 A_{0,1} B_{0,0} \psi^{(0)}(m+n+1)^2
\end{aligned}$$

$$\begin{aligned}
& + 3n^2 A_{0,0} B_{0,1} \psi^{(0)}(m+n+1)^2 \\
& + 3n^2 A_{0,0} B_{0,0} \psi^{(1)}(m+n+1) \psi^{(0)}(m+n+1) \\
& + 6n^2 A_{0,1} B_{0,1} \psi^{(0)}(m+n+1) \\
& + 9n^2 A_{0,0} B_{n,2} \psi^{(0)}(m+n+1) \\
& + n^2 A_{0,0} B_{0,0} \psi^{(2)}(m+n+1) + 2n^2 A_{0,1} B_{0,0} \psi^{(1)}(m+n+1) \\
& + 3n^2 A_{0,0} B_{0,1} \psi^{(1)}(m+n+1) \\
& + 4n A_{0,0} B_{0,0} \psi^{(0)}(m+n+1)^2 + 2A_{0,0} B_{0,0} \psi^{(0)}(m+n+1) \\
& + 8n A_{0,1} B_{0,0} \psi^{(0)}(m+n+1) + 6n A_{0,0} B_{0,1} \psi^{(0)}(m+n+1) \\
& + 4n A_{0,0} B_{0,0} \psi^{(1)}(m+n+1) + 18n^2 A_{0,1} B_{n,2} \\
& + 12n A_{0,1} B_{0,1} + 4A_{0,1} B_{0,0}), \\
\mathcal{A}_3^{(4)}(m, n) = & -\frac{2}{9} (18m A_{0,0} H_m B_{n,2} + 2m A_{0,0} B_{0,0} H_m \psi^{(0)}(m+n+1)^2 \\
& + 6m A_{0,0} B_{0,0} H_n \psi^{(0)}(m+n+1)^2 \\
& - 12m A_{0,0} B_{0,0} H_m H_n \psi^{(0)}(m+n+1) \\
& + 6A_{0,0} B_{0,0} H_n \psi^{(0)}(m+n+1) \\
& + 12m A_{0,1} B_{0,0} H_n \psi^{(0)}(m+n+1) \\
& + 6m A_{0,0} B_{0,1} H_m \psi^{(0)}(m+n+1) \\
& + 2m A_{0,0} B_{0,0} H_m \psi^{(1)}(m+n+1) \\
& + 6m A_{0,0} B_{0,0} H_n \psi^{(1)}(m+n+1) \\
& - 18m A_{0,1} B_{n,2} - m A_{0,0} B_{0,0} \psi^{(0)}(m+n+1)^3 \\
& - A_{0,0} B_{0,0} \psi^{(0)}(m+n+1)^2 - 2m A_{0,1} B_{0,0} \psi^{(0)}(m+n+1)^2 \\
& - 3m A_{0,0} B_{0,1} \psi^{(0)}(m+n+1)^2 \\
& - 3m A_{0,0} B_{0,0} \psi^{(1)}(m+n+1) \psi^{(0)}(m+n+1) \\
& - 3A_{0,0} B_{0,1} \psi^{(0)}(m+n+1) - 6m A_{0,1} B_{0,1} \psi^{(0)}(m+n+1) \\
& - 9m A_{0,0} B_{n,2} \psi^{(0)}(m+n+1) - A_{0,0} B_{0,0} \psi^{(1)}(m+n+1) \\
& - m A_{0,0} B_{0,0} \psi^{(2)}(m+n+1) - 2m A_{0,1} B_{0,0} \psi^{(1)}(m+n+1) \\
& - 3m A_{0,0} B_{0,1} \psi^{(1)}(m+n+1) - 9A_{0,0} B_{n,2}),
\end{aligned}$$

$$\mathcal{B}_1^{(1)}(m, n) = \frac{2}{9} n A_{0,0} B_{0,0} (m-n),$$

$$\mathcal{B}_2^{(1)}(m, n) = -\frac{2}{9} n A_{0,0} B_{0,0},$$

$$\mathcal{B}_3^{(1)}(m, n) = \frac{2}{9} A_{0,0} B_{0,0},$$

$$\mathcal{B}_1^{(2)}(m, n) = -\frac{1}{9}(m-n)(4nA_{0,0}B_{0,0}H_m + 6nA_{0,0}B_{0,0}H_n - 4nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1) - 4nA_{0,1}B_{0,0} - 3nA_{0,0}B_{0,1} - 2A_{0,0}B_{0,0}),$$

$$\mathcal{B}_2^{(2)}(m, n) = \frac{1}{9}(4nA_{0,0}B_{0,0}H_m + 6nA_{0,0}B_{0,0}H_n - 4nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1) - 4nA_{0,1}B_{0,0} - 3nA_{0,0}B_{0,1} - 2A_{0,0}B_{0,0}),$$

$$\mathcal{B}_3^{(2)}(m, n) = \frac{1}{9}(-4A_{0,0}B_{0,0}H_m - 6A_{0,0}B_{0,0}H_n + 4A_{0,0}B_{0,0}\psi^{(0)}(m+n+1) + 4A_{0,1}B_{0,0} + 3A_{0,0}B_{0,1}),$$

$$\begin{aligned} \mathcal{B}_1^{(3)}(m, n) = & \frac{1}{18}(-24n^2A_{0,0}B_{0,0}H_mH_n + 12n^2A_{0,0}B_{0,1}H_m \\ & + 8n^2A_{0,0}B_{0,0}H_m\psi^{(0)}(m+n+1) \\ & + 18n^2A_{0,0}B_{0,0}H_n\psi^{(0)}(m+n+1) + 16nA_{0,0}B_{0,0}H_m \\ & + 24mnA_{0,0}B_{0,0}H_mH_n - 24mnA_{0,1}B_{0,0}H_n \\ & - 12mnA_{0,0}B_{0,1}H_m - 6mA_{0,0}B_{0,0}H_n \\ & - 8mnA_{0,0}B_{0,0}H_m\psi^{(0)}(m+n+1) \\ & - 18mnA_{0,0}B_{0,0}H_n\psi^{(0)}(m+n+1) \\ & - 8mA_{0,0}B_{0,0}H_m + 24n^2A_{0,1}B_{0,0}H_n \\ & - 5n^2A_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 \\ & - 5n^2A_{0,0}B_{0,0}\psi^{(1)}(m+n+1) \\ & - 8n^2A_{0,1}B_{0,0}\psi^{(0)}(m+n+1) \\ & - 9n^2A_{0,0}B_{0,1}\psi^{(0)}(m+n+1) \\ & + 12mnA_{0,1}B_{0,1} + 9mnA_{0,0}B_{n,2} \\ & + 5mnA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 \\ & - 8nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1) \\ & + 5mnA_{0,0}B_{0,0}\psi^{(1)}(m+n+1) \\ & + 8mnA_{0,1}B_{0,0}\psi^{(0)}(m+n+1) \\ & + 9mnA_{0,0}B_{0,1}\psi^{(0)}(m+n+1) \\ & + 6mA_{0,0}B_{0,0}\psi^{(0)}(m+n+1) + 8mA_{0,1}B_{0,0} + 3mA_{0,0}B_{0,1} \\ & - 12n^2A_{0,1}B_{0,1} - 9n^2A_{0,0}B_{n,2} - 16nA_{0,1}B_{0,0} + 2A_{0,0}B_{0,0}), \end{aligned}$$

$$\mathcal{B}_2^{(3)}(m, n) = \frac{1}{18}(-24nA_{0,0}B_{0,0}H_mH_n + 12nA_{0,0}B_{0,1}H_m + 8nA_{0,0}B_{0,0}H_m\psi^{(0)}(m+n+1))$$

$$\begin{aligned}
& + 18nA_{0,0}B_{0,0}H_n\psi^{(0)}(m+n+1) \\
& + 8A_{0,0}B_{0,0}H_m + 6A_{0,0}B_{0,0}H_n + 24nA_{0,1}B_{0,0}H_n \\
& - 5nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 - 6A_{0,0}B_{0,0}\psi^{(0)}(m+n+1) \\
& - 8nA_{0,1}B_{0,0}\psi^{(0)}(m+n+1) - 9nA_{0,0}B_{0,1}\psi^{(0)}(m+n+1) \\
& - 5nA_{0,0}B_{0,0}\psi^{(1)}(m+n+1) - 12nA_{0,1}B_{0,1} - 9nA_{0,0}B_{n,2} \\
& - 8A_{0,1}B_{0,0} - 3A_{0,0}B_{0,1}),
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_3^{(3)}(m, n) = \frac{1}{18} & (24A_{0,0}B_{0,0}H_mH_n - 8A_{0,0}B_{0,0}H_m\psi^{(0)}(m+n+1) \\
& - 18A_{0,0}B_{0,0}H_n\psi^{(0)}(m+n+1) - 12A_{0,0}B_{0,1}H_m \\
& - 24A_{0,1}B_{0,0}H_n + 5A_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 \\
& + 8A_{0,1}B_{0,0}\psi^{(0)}(m+n+1) \\
& + 9A_{0,0}B_{0,1}\psi^{(0)}(m+n+1) + 5A_{0,0}B_{0,0}\psi^{(1)}(m+n+1) \\
& + 9A_{0,0}B_{n,2} + 12A_{0,1}B_{0,1}),
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_1^{(4)}(m, n) = \frac{1}{18} & (-n^2A_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^3 \\
& + mnA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^3 \\
& + 2mA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 \\
& - 3nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 \\
& + 2n^2H_mA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 \\
& - 2mnH_mA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 \\
& + 6n^2H_nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 \\
& - 6mnH_nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 \\
& - 2n^2A_{0,1}B_{0,0}\psi^{(0)}(m+n+1)^2 \\
& + 2mnA_{0,1}B_{0,0}\psi^{(0)}(m+n+1)^2 \\
& - 3n^2A_{0,0}B_{0,1}\psi^{(0)}(m+n+1)^2 \\
& + 3mnA_{0,0}B_{0,1}\psi^{(0)}(m+n+1)^2 \\
& - 4mH_mA_{0,0}B_{0,0}\psi^{(0)}(m+n+1) \\
& + 8nH_mA_{0,0}B_{0,0}\psi^{(0)}(m+n+1) \\
& - 6mH_nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1) \\
& + 6nH_nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1) \\
& - 12n^2H_mH_nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1) \\
& + 12mnH_mH_nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)
\end{aligned}$$

$$\begin{aligned}
& -3n^2\psi^{(1)}(m+n+1)A_{0,0}B_{0,0}\psi^{(0)}(m+n+1) \\
& +3mn\psi^{(1)}(m+n+1)A_{0,0}B_{0,0}\psi^{(0)}(m+n+1) \\
& +4mA_{0,1}B_{0,0}\psi^{(0)}(m+n+1) - 8nA_{0,1}B_{0,0}\psi^{(0)}(m+n+1) \\
& +12n^2H_nA_{0,1}B_{0,0}\psi^{(0)}(m+n+1) \\
& -12mnH_nA_{0,1}B_{0,0}\psi^{(0)}(m+n+1) \\
& +3mA_{0,0}B_{0,1}\psi^{(0)}(m+n+1) - 3nA_{0,0}B_{0,1}\psi^{(0)}(m+n+1) \\
& +6n^2H_mA_{0,0}B_{0,1}\psi^{(0)}(m+n+1) \\
& -6mnH_mA_{0,0}B_{0,1}\psi^{(0)}(m+n+1) \\
& -6n^2A_{0,1}B_{0,1}\psi^{(0)}(m+n+1) \\
& +6mnA_{0,1}B_{0,1}\psi^{(0)}(m+n+1) \\
& -9n^2A_{0,0}B_{n,2}\psi^{(0)}(m+n+1) \\
& +9mnA_{0,0}B_{n,2}\psi^{(0)}(m+n+1) \\
& +4H_mA_{0,0}B_{0,0} + 12mH_mH_nA_{0,0}B_{0,0} - 24nH_mH_nA_{0,0}B_{0,0} \\
& -6H_nA_{0,0}B_{0,0} + 2m\psi^{(1)}(m+n+1)A_{0,0}B_{0,0} \\
& -3n\psi^{(1)}(m+n+1)A_{0,0}B_{0,0} \\
& +2n^2H_m\psi^{(1)}(m+n+1)A_{0,0}B_{0,0} \\
& -2mnH_m\psi^{(1)}(m+n+1)A_{0,0}B_{0,0} \\
& +6n^2H_n\psi^{(1)}(m+n+1)A_{0,0}B_{0,0} \\
& -6mnH_n\psi^{(1)}(m+n+1)A_{0,0}B_{0,0} \\
& -n^2\psi^{(2)}(m+n+1)A_{0,0}B_{0,0} + mn\psi^{(2)}(m+n+1)A_{0,0}B_{0,0} \\
& -12mH_nA_{0,1}B_{0,0} + 24nH_nA_{0,1}B_{0,0} \\
& -2n^2\psi^{(1)}(m+n+1)A_{0,1}B_{0,0} \\
& +2mn\psi^{(1)}(m+n+1)A_{0,1}B_{0,0} \\
& -4A_{0,1}B_{0,0} - 6mH_mA_{0,0}B_{0,1} \\
& +12nH_mA_{0,0}B_{0,1} - 3n^2\psi^{(1)}(m+n+1)A_{0,0}B_{0,1} \\
& +3mn\psi^{(1)}(m+n+1)A_{0,0}B_{0,1} + 3A_{0,0}B_{0,1} + 6mA_{0,1}B_{0,1} \\
& -12nA_{0,1}B_{0,1} + 9nA_{0,0}B_{n,2} + 18n^2H_mA_{0,0}B_{n,2} \\
& -18mnH_mA_{0,0}B_{n,2} - 18n^2A_{0,1}B_{n,2} + 18mnA_{0,1}B_{n,2}),
\end{aligned}$$

$$\begin{aligned}
\mathcal{B}_2^{(4)}(m, n) &= \frac{1}{18} \left( -12A_{0,0}B_{0,0}H_mH_n + 18nA_{0,0}H_mB_{n,2} \right. \\
& \quad \left. + 2nA_{0,0}B_{0,0}H_m\psi^{(0)}(m+n+1)^2 \right. \\
& \quad \left. + 6nA_{0,0}B_{0,0}H_n\psi^{(0)}(m+n+1)^2 \right)
\end{aligned}$$

$$\begin{aligned}
& + 4A_{0,0}B_{0,0}H_m\psi^{(0)}(m+n+1) \\
& - 12nA_{0,0}B_{0,0}H_mH_n\psi^{(0)}(m+n+1) \\
& + 6A_{0,0}B_{0,0}H_n\psi^{(0)}(m+n+1) \\
& + 12nA_{0,1}B_{0,0}H_n\psi^{(0)}(m+n+1) \\
& + 6nA_{0,0}B_{0,1}H_m\psi^{(0)}(m+n+1) \\
& + 2nA_{0,0}B_{0,0}H_m\psi^{(1)}(m+n+1) \\
& + 6nA_{0,0}B_{0,0}H_n\psi^{(1)}(m+n+1) \\
& + 6A_{0,0}B_{0,1}H_m + 12A_{0,1}B_{0,0}H_n \\
& - nA_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^3 \\
& - 2A_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^2 - 2nA_{0,1}B_{0,0}\psi^{(0)}(m+n+1)^2 \\
& - 3nA_{0,0}B_{0,1}\psi^{(0)}(m+n+1)^2 \\
& - 3nA_{0,0}B_{0,0}\psi^{(1)}(m+n+1)\psi^{(0)}(m+n+1) \\
& - 4A_{0,1}B_{0,0}\psi^{(0)}(m+n+1) - 3A_{0,0}B_{0,1}\psi^{(0)}(m+n+1) \\
& - 6nA_{0,1}B_{0,1}\psi^{(0)}(m+n+1) - 9nA_{0,0}B_{n,2}\psi^{(0)}(m+n+1) \\
& - 2A_{0,0}B_{0,0}\psi^{(1)}(m+n+1) - nA_{0,0}B_{0,0}\psi^{(2)}(m+n+1) \\
& - 2nA_{0,1}B_{0,0}\psi^{(1)}(m+n+1) - 3nA_{0,0}B_{0,1}\psi^{(1)}(m+n+1) \\
& - 18nA_{0,1}B_{n,2} - 6A_{0,1}B_{0,1}), \\
\mathcal{B}_3^{(4)}(m,n) = & \frac{1}{18}(-18A_{0,0}H_mB_{n,2} - 2A_{0,0}B_{0,0}H_m\psi^{(0)}(m+n+1)^2 \\
& - 6A_{0,0}B_{0,0}H_n\psi^{(0)}(m+n+1)^2 \\
& + 12A_{0,0}B_{0,0}H_mH_n\psi^{(0)}(m+n+1) \\
& - 12A_{0,1}B_{0,0}H_n\psi^{(0)}(m+n+1) \\
& - 6A_{0,0}B_{0,1}H_m\psi^{(0)}(m+n+1) \\
& - 2A_{0,0}B_{0,0}H_m\psi^{(1)}(m+n+1) \\
& - 6A_{0,0}B_{0,0}H_n\psi^{(1)}(m+n+1) + A_{0,0}B_{0,0}\psi^{(0)}(m+n+1)^3 \\
& + 2A_{0,1}B_{0,0}\psi^{(0)}(m+n+1)^2 + 3A_{0,0}B_{0,1}\psi^{(0)}(m+n+1)^2 \\
& + 3A_{0,0}B_{0,0}\psi^{(1)}(m+n+1)\psi^{(0)}(m+n+1) \\
& + 6A_{0,1}B_{0,1}\psi^{(0)}(m+n+1) \\
& + 9A_{0,0}B_{n,2}\psi^{(0)}(m+n+1) + A_{0,0}B_{0,0}\psi^{(2)}(m+n+1) \\
& + 2A_{0,1}B_{0,0}\psi^{(1)}(m+n+1) + 3A_{0,0}B_{0,1}\psi^{(1)}(m+n+1) \\
& + 18A_{0,1}B_{n,2}).
\end{aligned}$$