LAKE COMO SCHOOL OF ADVANCED STUDIES

Partial Differential Equations of Mathematical Physics and Applications

Italy - Como, 13-17 September 2021

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Main Speakers

Giovanni Paolo Galdi – University of Pittsburgh

Alain Haraux – Université Pierre et Marie Curie, Paris

Walter Lacarbonara – Sapienza Università di Roma

Scientific Committee

Maurizio Garrione, Filippo Gazzola

Organizing Committee

Alessandra Cazzaniga, Alessia Adamo

Invited Speakers

Elvise Berchio (Torino), Denis Bonheure (Bruxelles), Francesca Colasuonno (Bologna), Alberto Ferrero (Piemonte Orientale), Céline Grandmont (Paris), Gabriela Holubová (Plzeň), Paolo Maremonti (Campania), Andrei Metrikine (Delft), Šárka Nečasová (Praha), Sebastian Schwarzacher (Praha), Axelle Viré (Delft)

Sponsors

- ARC Advanced 2020-2025 Partial Differential Equations in interaction - funded by Fédération Wallonie-Bruxelles at ULB
- Department of Mathematics and University Research Funds - Politecnico di Milano
- PRIN 2017 Direct and inverse problems for partial differential equations: theoretical aspects and applications
- GNAMPA-INdAM Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni - Roma

Villa del Grumello
Via per Cernobbio 11
22100 Como, Italy
Tel. +39.031.579813
Fax +39.031.573395
E-mail: info@lakecomoschool.org
www.lakecomoschool.org

Preface

By typing the words "fluid" and "structure" in the field "anywhere" of the *Mathscinet* database, and by selecting the dates in the field "year", it appears evident how fluid-structure interactions is becoming more and more important in mathematical researches. In fact, besides the physical phenomenon of the mutual interaction between fluids and structures, there are also several common features in the mathematical models (PDEs) governing elasticity and fluid mechanics. This is why we had the idea of bringing together specialists from both fields and to collect their contributions in this volume on *Interactions between elasticity and fluid mechanics*. Some of these specialists were present at a Summer School that we organized in the framework of the *Lake Como School of Advanced Studies* in September 2021.[1] In that occasion, we decided to collect some of the recent researches of the invited speakers and to complement them with some contributions of other colleagues.

Chapter 1 provides a complete account on the method of adapted energies for damped and forced Duffing-type equations, both in finite and infinite dimension, showing its applications in establishing stability and asymptotic stability results, as well as ultimate bounds on the solutions. The final part of the chapter focuses on phenomena on large time scales for damped Kepler-like problems, which are put in relation with some atomic models. The author debates about some concrete observations of physical phenomena proposing, among the other considerations, a suggestive hypothesis of atomic contraction. Overall, the contribution turns out to be a useful report about the Duffing equation, which frequently arises in models describing the dynamics of suspension bridges; a series of valuable results, helping to better understand the behavior of the corresponding solutions, are provided, together with their detailed proofs coming with all the relevant computations.

Chapter 2 deals with a nonlinear system of two coupled oscillators, describing the vertical and the angular displacement of a rod, representing the cross section of the deck of a suspension bridge, subject to a nonlinear restoring force exerted by the hangers. By means of a computer assisted proof, the author determines the bifurcation diagram for the periodic solutions, in dependence of the amplitude of the vertical oscillations. It is shown that a pitchfork bifurcation takes place, beyond which the vertical oscillations become unstable while the torsional ones are stable. This represents an explanation of the onset of large torsional motions in a suspension bridge, as the result of a relevant energy transfer from the vertical to the torsional oscillations.

[1] See the poster of the event at the beginning of the volume.

In Chapter 3, the authors deal with a stationary plate model for a suspension bridge, investigating a measure of the torsional displacement of the structure called *gap function*. They consider impulsive-type external loads and they are interested in the *worst-case* problem of maximizing the gap function with respect to the location of such impulses, in order to spot the external forces which produce the highest risk for the structure. Through an explicit computation of the gap function, they show that the most dangerous loads in a suitably restricted class are the odd ones acting in the midpoints of the long edges, giving a contribution towards the solution of a previous conjecture.

Chapter 4 develops in detail an orthotropic plate model for the deck of a suspension bridge. The main novelty with respect to the usual plate models is to admit different elastic responses according to the direction, by allowing the rigidity in the sense of the length to be different from the one in the sense of the width. After a detailed review of the physical background, the author derives the stationary equation by a variational principle, and then discusses the related eigenvalue problem, making a comparison with the isotropic plate case. This represents the starting point for the study of evolutive orthotropic models for the dynamics of a suspension bridge, taking into account the tension of the cables and the restoring forces exerted by the hangers.

Chapter 5 deals with a purely theoretical problem in fluid dynamics, with no structure. A Cauchy problem for the Navier–Stokes equations in $\mathbb{R}^3$ is analyzed for solenoidal initial data in $L^3(\mathbb{R}^3)$. A new sufficient condition on the data is provided in order to determine the time interval of existence. This is the mandatory step before studying related problems in bounded domains.

Chapter 6 deals with a typical problem in fluid-structure interaction models. A compressible isentropic fluid which contains several rigid bodies is considered, and a local-in-time existence of a weak solution for the associated system of partial differential equations is obtained. The fluid-structure interaction is incorporated within the Navier-slip boundary condition at the interface of the fluid and the rigid bodies. On the remaining part of the boundary of the container, the fluid is assumed to satisfy a no-slip (Dirichlet) condition. The novelties comprise a new bound on the maximal time for which existence of weak solutions is ensured.

Chapter 7 makes a strong connection between abstract theory and technical practice. Some dynamical systems associated to suitable fluid-structure interaction problems exhibit a global attractor, showing both the dissipativity of the system and the possibility to forecast its long-term dynamics. But what is really present in the attractor may be difficult to characterize. The main novelty of this contribution is to numerically approximate the objects within the attractor. The model considered is the air interacting with the deck of a bridge. The wind-induced motions of the deck of a bridge are modeled as in a wind tunnel experiment.

Chapter 8 considers nonlinear models for suspension bridges, where the structure is composed by two cables, source of geometric nonlinearity, hangers, and a beam-like deck-girder. First, a planar model for symmetric bridges undergoing loads preserving the symmetry is considered. Second, a three-dimensional model taking into account the full geometric nonlinearities in the structure is proposed. For both models, a system of partial differential equations comprising all the mechanical parameters is obtained, and some real cases of suspension bridges are discussed in order to support the numerical results.

Chapter 9 discusses the role of symmetry in stability issues for both fluids and structures. It turns out that in many situations, symmetry stabilizes fluids, for instance, flows with small Reynolds number, whereas symmetry breaking leads to instability. On the other hand, for some structures, such as plates and beams modeling suspension bridges, symmetry may play against stability.

Milan, November 2022 Maurizio Garrione and Filippo Gazzola