Preface

Avant donc que d'écrire, apprenez à penser.
Selon que notre idée est plus ou moins obscure,
L'expression la suit, ou moins nette, ou plus pure.
Ce que l'on conçoit bien s'énonce clairement,
Et les mots pour le dire arrivent aisément.
[...]
Hâtez-vous lentement, et, sans perdre courage,
Vingt fois sur le métier remettez votre ouvrage
Polissez-le sans cesse et le repolissez ;
Ajoutez quelquefois, et souvent effacez.
[...]
Il faut que chaque chose y soit mise en son lieu ;
Que le début, la fin, répondent au milieu ;
Que d'un art délicat les pièces assorties
N'y forment qu'un seul tout de diverses parties,
Que jamais du sujet le discours s'écartant
N'aille chercher trop loin quelque mot éclatant.

Boileau, L'art poétique, chant I (1674)

Learn then to Think, e'er you pretend to Write,
As your Idea's clear, or else obscure,
Th'expression follows perfect, or impure:
What we conceive, with ease we can express;
Words to the Notions flow with readiness.
[...]
Gently make hast, of Labour not afraid;
A hundred times consider what you've said;
Polish, repolish, every Colour lay,
And sometimes add; but oft'ner take away.
[...]
Each Object must be fix'd in the due place,
And diff'ring parts have Corresponding Grace:
Till by a curious Art dispos'd we find
One perfect whole, of all the pieces joyn'd.
Keep to your Subject close in all you say;
Nor for a sounding Sentence ever stray.

Boileau, The Art of Poetry, Canto I (1674)
Made English by Soames (1680)
Revis'd by Dryden (1683)

Graph theory has been one of the most fertile fields in mathematics in the past fifty years. This undoubtedly comes from the huge field of application of the combinatorial structures it deals with, as well as from the need to discretise many results in mathematics connected to computer science and also to quantum physics.

The mathematical field of graph theory may be said to have begun with an article by the Swiss mathematician Leonhard Euler on the famous Königsberg bridge problem [13]. Nevertheless, although there have been many uses of graphs, already in the Middle Ages [25], it will be necessary to wait until the 20th century before the publication of the first book on the subject, written by Dénes König [23].

The development of graph theory is quite similar to the development of probability theory, in which many results are due to an effort to understand how games of chance work. Graphs were first considered as mathematical curiosities, mere tools to use in the study of logic puzzles, such as the classical problem of whether the knight on a chessboard is able to visit every square exactly once and return to its original position.

However, graphs have now become indispensable in many areas of human endeavour. Following Euler, an increasing number of mathematicians grew interested in graphs. Let us name a few:

- In the 19th century, Edouard Lucas invented the following problem: An even number of girls walk out each day two by two; is there any way to organise their walks in such a way that each of them is paired exactly once with each of the others? The tools used to solve this type of problems are developed in Chapter 8 of this book.

- In 1856, William R. Hamilton studied a problem that appeared just as simple: Find a path that passes exactly once through every vertex of a graph. This problem gave rise to the concept of a Hamiltonian graph (see Chapter 2).
- In the 19th century, Gustav Kirchhoff used implicit graph-theoretical tools to study currents in electrical circuits (see Chapter 4).
- In the 19th century, Arthur Cayley, James J. Sylvester, and then, in the 20th century, George Pólya developed the notion of a tree in order to enumerate chemical compounds (see Chapter 2).
- In 1840, August F. Möbius stated the following problem: A king with five sons wishes for his kingdom to be divided into five provinces at his death, in such a way that each province has at least one common border with each of the others. This problem comes down to deciding whether the complete graph with five vertices K_5 is planar; the concept of planar graphs is introduced in Chapter 5.

One of the main difficulties in graph theory is that the language and notation are not yet fully established; sometimes they come directly from the field in which some aspect of graph theory was first employed.

We have tried throughout the book to unify definitions and results and place them in the most general framework possible. However, it is not always easy to manage using the same terminology for simple graphs, multigraphs, directed graphs (or digraphs) and so on.

In order to provide a self-contained reading, we have endeavoured to give detailed proofs as well as introduce mathematical notions throughout the book. We have paid attention to the algebraic and topological aspect of graph theory, these often being treated concisely in classical textbooks. In addition, we also included a chapter on random graphs as well as another on analysis applied to graphs which allowed us to give applications to some physical theories.

This book aims to take a person with very little knowledge of graphs and bring him or her at the threshold of research. Consequently, it has a wide potential readership, ranging from students of all levels to established researchers.

The original edition of this book appeared in 2012 under the title *Eléments de théorie des graphes*. Its second edition, still written in French, appeared in 2018 under the same title and contains additional developments on the spectral theory of graphs, random graphs and weighted graphs. The present English edition is based on the enlarged second French edition.

We hope that this work will be useful to anyone wishing to study graph theory, both from the mathematical perspective and from a computer science point of view, keeping in mind that many of the chapters deal with graph algorithms.

For further reading, a short list of classical texts on graphs can be found at the end of the book.

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