Leibniz and the Brachistochrone

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1696 was the year of birth of the calculus of variations. As usual in those days, the Swiss mathematician Johann Bernoulli, one of Leibniz's closest friends and followers, issued a provocative mathematical challenge in the scholarly journal Acta Eruditorum (Transactions of scholars) in June 1696 inviting the mathematicians to solve this new problem:

> *Given two points A and B in a vertical plane, find the path AMB down which a movable point M must by virtue of its weight fall from A to B in the shortest possible time.*

In order to encourage "the enthusiasts of such things" (harum rerum amatores) Bernoulli emphasized the usefulness of the problem not only in mechanics but also in other sciences and added that the curve being sought is not the straight line but a curve well-known to geometers. He would publicize it by the end of the year if nobody should publicize it within this period. When Bernoulli published his challenge he did not know that Galileo had dealt with a related problem without having in mind Bernoulli's generality. And he could not know that his challenge would lead to one of the most famous priority disputes in the history of mathematics.

He communicated the problem to Leibniz in a private letter, dated June 19, 1696 and dispatched from Groningen in the Netherlands, asking him to occupy himself with it. Leibniz sent him his answer, together with the correct solution, just one week later on June 26 from Hannover. He proposed the name tachystoptota (curve of quickest descent), avowing that the problem is indeed most beautiful and that it had attracted him against his will and that he hesitated because of its beauty like Eve before the apple. He deduced the correct differential equation but failed to recognize that the curve was a cycloid until Bernoulli informed him in his answer dating from July 31. He took up Leibniz's biblical reference adding that he was very happy about this comparison provided that he was not regarded as the snake that had offered the apple. Leibniz must certainly have been happy to hear that the curve is the

Figure 1: Bernoulli's figure of the brachistochrone (Die Streitschriften von Jacob und Johann Bernoulli, Variationsrechnung. Bearbeitet und kommentiert von Herman H. Goldstine, mit historischen Anmerkungen von Patricia Radelet-de Grave. Basel-Boston-Berlin 1991, 212)

cycloid, for which Huygens had shown the property of isochronism. For that reason he, Bernoulli, had given it the name brachystochrona. Leibniz adopted Bernoulli's description.

On June 28 he had already communicated the problem to Rudolf Christian von Bodenhausen in Florence, again praising its extraordinary beauty in order to encourage the Italian mathematicians to solve it. In Switzerland Jacob Bernoulli, and in France Pierre Varignon, had been informed. He asked Johann Bernoulli to extend the deadline until June 1697 because in late autumn 1696 the existence of only three solutions, by Johann and his elder brother Jacob Bernoulli and by himself, were known. Bernoulli agreed insofar as he published a new announcement in the December issue of the Acta Eruditorum that he would suppress his own solution until Easter 1697. In addition to that he wrote a printed leaflet that appeared in January 1697.

The May 1697 issue of the Acta Eruditorum contained an introductory historical paper by Leibniz on the catenary and on the brachistochrone. He renounced the publication of his own solution of the brachistochrone problem because it corresponded, he said, with the other solutions (cum caeteris consentiat). Then the five known solutions by Johann, Jacob Bernoulli, the Marquis de l'Hospital, Ehrenfried Walther von Tschirnhaus, and Isaac Newton were published or reprinted (Newton). Newton had not revealed his name. But Johann Bernoulli recognized the author, "from the claw of the lion" (ex ungue leonem), as he said.

Figure 2: Galileo's figure regarding the fall of a particle along a circular polygon (Galileo Galilei: Le opere, vol. VIII, Firenze 1965, 262)

Leibniz made some statements in his paper that are worth discussing. First of all he maintained that Galileo had already dealt with the catenary and with the brachistochrone as well, without being able to find the correct solution. He had falsely identified the catenary with a parabola and the brachistochrone with a circular arc. Unfortunately Johann Bernoulli relied on Leibniz's false statement and repeated it in June 1697, and later so did many other authors up to the present time. Neither the one nor the other assertion is in reality true. What had Galileo really said in his Discorsi? He had rightly emphasized the similarity between the catenary and a parabola. He did not and could not look for the curve of quickest descent, that is, for the brachistochrone. Such a general problem was still beyond the mathematical horizon of the mathematicians of his time.

He had considered an arc of a circle CBD of not more than 90° in a vertical plane with C the lowest point on the circle, D the highest point and B any other point on the arc of the circle. He proved the correct theorem that the time for a particle to fall along the broken line DBC is less than the time for it to descend along the line DC. Let us enlarge the number of points on the circle between D and C. The larger the number of points is, the less is the time for the particle to descend along the broken line $DEFG \dots C$. For Galileo a circle was a polygon with infinitely many, infinitely small sides. Hence he rightly concluded that the swiftest time of fall from D to C is along a portion of the circle. Galileo only compared the times of fall along the sides of circular polygons the circle being the limit case of them.

Secondly, Leibniz said that the only mathematicians to have solved the problem are those he had guessed would be capable of solving it; in other words, only those who had suffiently penetrated in the mysteries of his differential calculus. This he had predicted for the brother of Johann Bernoulli and the Marquis de l'Hospital, for Huygens if he were alive, for Hudde if he had not given up such pursuits, for Newton if he would take the trouble. The words were carelessly written because their obvious meaning was that Newton was indebted to the differential calculus for his solution. Even if Leibniz did not want to make such a claim, and this is certain in 1697, his words could be interpreted in such a way. There was indeed a reader who chose this interpretation: the French emigrant Nicolas Fatio de Duillier, one of Newton's closest followers. Fatio was deeply offended at not having been mentioned by Leibniz among those authors who could master the brachistochrone problem. In 1699 he published a lengthy analysis of the brachistochrone. Therein he praised his own mathematical originality and sharply accused Leibniz of being only the second inventor of the calculus. Fatio's publication was the beginning of the calculus squabble. But this is another story.

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