

Abstract

The main focus in this memoir is on Laplacians on both weighted graphs and weighted metric graphs. Let us emphasize that we consider infinite locally finite graphs and do not make any further geometric assumptions. Whereas the existing literature usually treats these two types of Laplacian operators separately, we approach them in a uniform manner in the present work and put particular emphasis on the relationship between them. One of our main conceptual messages is that these two settings should be regarded as complementary (rather than opposite) and exactly their interplay leads to important further insight on both sides. Our central goal is twofold. First of all, we explore the relationships between these two objects by comparing their basic spectral (self-adjointness, spectral gap, etc.), parabolic (Markovian uniqueness, recurrence, stochastic completeness, etc.), and metric (quasi-isometries, intrinsic metrics, etc.) properties. In turn, we exploit these connections either to prove new results for Laplacians on metric graphs or to provide new proofs and perspective on the recent progress in weighted graph Laplacians. We also demonstrate our findings by considering several important classes of graphs (Cayley graphs, tessellations, and antitrees).

Keywords. Graph, metric graph, Laplacians on graphs, spectral graph theory, self-adjointness, Markovian uniqueness, spectral gap, recurrence, ultracontractivity

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