

## References

- [1] S. I. Adyan, Random walks on free periodic groups. *Izv. Akad. Nauk SSSR Ser. Mat.* **46** (1982), no. 6, 1139–1149
- [2] N. I. Akhiezer, *The classical moment problem and some related questions in analysis*. Hafner Publishing Co., New York, 1965
- [3] S. Albeverio, F. Gesztesy, R. Høegh-Krohn, and H. Holden, *Solvable models in quantum mechanics*. 2nd edn., AMS Chelsea Publishing, Providence, RI, 2005
- [4] S. Albeverio, A. Kostenko, and M. Malamud, Spectral theory of semibounded Sturm–Liouville operators with local interactions on a discrete set. *J. Math. Phys.* **51** (2010), no. 10, art. 102102
- [5] N. Alon, Eigenvalues and expanders. *Combinatorica* **6** (1986), 83–96
- [6] N. Alon and V. D. Milman,  $\lambda_1$ , isoperimetric inequalities for graphs, and superconcentrators. *J. Combin. Theory Ser. B* **38** (1985), no. 1, 73–88
- [7] C. Anné and N. Torki-Hamza, The Gauss–Bonnet operator of an infinite graph. *Anal. Math. Phys.* **5** (2015), no. 2, 137–159
- [8] A. I. Aptekarev, S. A. Denisov, and M. L. Yattselev, Self-adjoint Jacobi matrices on trees and multiple orthogonal polynomials. *Trans. Amer. Math. Soc.* **373** (2020), no. 2, 875–917
- [9] A. I. Aptekarev, S. A. Denisov, and M. L. Yattselev, Jacobi matrices on trees generated by Angelesco systems: Asymptotics of coefficients and essential spectrum. *J. Spectr. Theory* **11** (2021), no. 4, 1511–1597
- [10] N. Avni, J. Breuer, and B. Simon, Periodic Jacobi matrices on trees. *Adv. Math.* **370** (2020), art. 107241
- [11] M. Baker and R. Rumely, *Potential theory and dynamics on the Berkovich projective line*. Math. Surveys Monogr. 159, American Mathematical Society, Providence, RI, 2010
- [12] M. T. Barlow, *Random walks and heat kernels on graphs*. London Math. Soc. Lecture Note Ser. 438, Cambridge University Press, Cambridge, 2017
- [13] M. T. Barlow and R. F. Bass, Stability of parabolic Harnack inequalities. *Trans. Amer. Math. Soc.* **356** (2004), no. 4, 1501–1533
- [14] M. T. Barlow, R. F. Bass, and T. Kumagai, Stability of parabolic Harnack inequalities on metric measure spaces. *J. Math. Soc. Japan* **58** (2006), no. 2, 485–519
- [15] M. T. Barlow and M. Murugan, Stability of the elliptic Harnack inequality. *Ann. of Math.* (2) **187** (2018), no. 3, 777–823
- [16] L. Bartholdi and B. Virág, Amenability via random walks. *Duke Math. J.* **130** (2005), no. 1, 39–56
- [17] F. Baudoin and D. J. Kelleher, Differential one-forms on Dirichlet spaces and Bakry–Émery estimates on metric graphs. *Trans. Amer. Math. Soc.* **371** (2019), no. 5, 3145–3178

- [18] F. Bauer, M. Keller, and R. K. Wojciechowski, Cheeger inequalities for unbounded graph Laplacians. *J. Eur. Math. Soc. (JEMS)* **17** (2015), no. 2, 259–271
- [19] O. Baues and N. Peyerimhoff, Curvature and geometry of tessellating plane graphs. *Discrete Comput. Geom.* **25** (2001), no. 1, 141–159
- [20] J. R. Baxter and R. V. Chacon, The equivalence of diffusions on networks to Brownian motion. In *Conference in modern analysis and probability (New Haven, Conn., 1982)*, pp. 33–48, Contemp. Math. 26, Amer. Math. Soc., Providence, RI, 1984
- [21] I. Benjamini and O. Schramm, Harmonic functions on planar and almost planar graphs and manifolds, via circle packings. *Invent. Math.* **126** (1996), no. 3, 565–587
- [22] I. Benjamini and O. Schramm, Random walks and harmonic functions on infinite planar graphs using square tilings. *Ann. Probab.* **24** (1996), no. 3, 1219–1238
- [23] Yu. M. Berezanski, *Selfadjoint operators in spaces of functions of infinitely many variables*. Transl. Math. Monographs 63, American Mathematical Society, Providence, RI, 1986
- [24] G. Berkolaiko, R. Carlson, S. A. Fulling, and P. Kuchment, *Quantum graphs and their applications*. Contemp. Math. 415, American Mathematical Society, Providence, RI, 2006
- [25] G. Berkolaiko and P. Kuchment, *Introduction to quantum graphs*. Math. Surveys Monogr. 186, American Mathematical Society, Providence, RI, 2013
- [26] R. Bessonov and S. Denisov, A spectral Szegő theorem on the real line. *Adv. Math.* **359** (2020), art. 106851
- [27] M. Biagioli and U. Mosco, A Saint–Venant type principle for Dirichlet forms on discontinuous media. *Ann. Mat. Pura Appl. (4)* **169** (1995), 125–181
- [28] B. Bollobás, *Graph theory*. Grad. Texts in Math. 63, Springer, New York, 1979
- [29] M. Braverman, O. Milatovic, and M. Shubin, Essential self-adjointness of Schrödinger-type operators on manifolds. *Russ. Math. Surveys* **57** (2002), no. 4(346), 641–692
- [30] J. Breuer and M. Keller, Spectral analysis of certain spherically homogeneous graphs. *Oper. Matrices* **7** (2013), no. 4, 825–847
- [31] J. Breuer and N. Levi, On the decomposition of the Laplacian on metric graphs. *Ann. Henri Poincaré* **21** (2020), no. 2, 499–537
- [32] H. Brezis, *Functional analysis, Sobolev spaces and partial differential equations*. Universitext, Springer, New York, 2011
- [33] R. Brooks, The fundamental group and the spectrum of the Laplacian. *Comment. Math. Helv.* **56** (1981), no. 4, 581–598
- [34] R. Brooks, A relation between growth and the spectrum of the Laplacian. *Math. Z.* **178** (1981), no. 4, 501–508
- [35] J. Brüning, V. Geyler, and K. Pankrashkin, Spectra of self-adjoint extensions and applications to solvable Schrödinger operators. *Rev. Math. Phys.* **20** (2008), no. 1, 1–70
- [36] A. Brzoska, C. George, S. Jarvis, L. G. Rogers, and A. Teplyaev, Spectral properties of graphs associated to the Basilica group. 2019, arXiv:[1908.10505](https://arxiv.org/abs/1908.10505)

- [37] D. Burago, Y. Burago, and S. Ivanov, *A course in metric geometry*. Grad. Stud. Math. 33, American Mathematical Society, Providence, RI, 2001
- [38] P. Buser, A note on the isoperimetric constant. *Ann. Sci. Éc. Norm. Supér. (4)* **15** (1982), no. 2, 213–230
- [39] E. A. Carlen, S. Kusuoka, and D. W. Stroock, Upper bounds for symmetric Markov transition functions. *Ann. Inst. H. Poincaré Probab. Stat.* **23** (1987), no. 2, 245–287
- [40] J. Carmesin and A. Georgakopoulos, Every planar graph with the Liouville property is amenable. *Random Structures Algorithms* **57** (2020), no. 3, 706–729
- [41] C. Cattaneo, The spectrum of the continuous Laplacian on a graph. *Monatsh. Math.* **124** (1997), no. 3, 215–235
- [42] J. Cheeger, A lower bound for the smallest eigenvalue of the Laplacian. In *Problems in analysis (Papers dedicated to Salomon Bochner, 1969)*, pp. 195–199, Princeton University Press, Princeton, N.J., 1970
- [43] F. R. K. Chung, *Spectral graph theory*. CBMS Reg. Conf. Ser. Math. 92, American Mathematical Society, Providence, RI, 1997
- [44] Y. Colin de Verdière, *Spectres de graphes*. Cours Spéc. 4, Société Mathématique de France, Paris, 1998
- [45] Y. Colin de Verdière, N. Torki-Hamza, and F. Truc, Essential self-adjointness for combinatorial Schrödinger operators II—metrically non complete graphs. *Math. Phys. Anal. Geom.* **14** (2011), no. 1, 21–38
- [46] T. Coulhon, Ultracontractivity and Nash type inequalities. *J. Funct. Anal.* **141** (1996), no. 2, 510–539
- [47] T. Coulhon and L. Saloff-Coste, Variétés riemanniennes isométriques à l’infini. *Rev. Mat. Iberoam.* **11** (1995), no. 3, 687–726
- [48] D. Cushing, S. Liu, F. Münch, and N. Peyerimhoff, Curvature calculations for antitrees. In *Analysis and geometry on graphs and manifolds*, pp. 21–54, London Math. Soc. Lecture Note Ser. 461, Cambridge University Press, Cambridge, 2020
- [49] D. Damanik, L. Fang, and S. Sukhtaiev, Zero measure and singular continuous spectra for quantum graphs. *Ann. Henri Poincaré* **21** (2020), no. 7, 2167–2191
- [50] N.-B. Dang, R. Grigorchuk, and M. Lyubich, Self-similar groups and holomorphic dynamics: renormalization, integrability and spectrum. *Arnold Math. J.* (2023), DOI 10.1007/s40598-022-00223-0
- [51] E. B. Davies, *Heat kernels and spectral theory*. Cambridge Tracts in Math. 92, Cambridge University Press, Cambridge, 1989
- [52] E. B. Davies, Analysis on graphs and noncommutative geometry. *J. Funct. Anal.* **111** (1993), no. 2, 398–430
- [53] E. B. Davies, Large deviations for heat kernels on graphs. *J. Lond. Math. Soc. (2)* **47** (1993), no. 1, 65–72
- [54] P. de la Harpe, *Topics in geometric group theory*. Chic Lectures Math., University of Chicago Press, Chicago, IL, 2000

- [55] V. A. Derkach and M. M. Malamud, Generalized resolvents and the boundary value problems for Hermitian operators with gaps. *J. Funct. Anal.* **95** (1991), no. 1, 1–95
- [56] V. A. Derkach, and M. M. Malamud, *The theory of extensions of symmetric operators and boundary value problems* (in Russian). Proc. Inst. Math. 104, NAS of Ukraine, Kiev, 2017
- [57] M. DeVos and B. Mohar, An analogue of the Descartes–Euler formula for infinite graphs and Higuchi’s conjecture. *Trans. Amer. Math. Soc.* **359** (2007), no. 7, 3287–3300
- [58] R. Diestel, *Graph theory*. 5th edn., Grad. Texts in Math. 173, Springer, Heidelberg, 2017
- [59] J. Dodziuk, Difference equations, isoperimetric inequality and transience of certain random walks. *Trans. Amer. Math. Soc.* **284** (1984), no. 2, 787–794
- [60] J. Dodziuk and L. Karp, Spectral and function theory for combinatorial Laplacians. In *Geometry of random motion (Ithaca, N.Y., 1987)*, pp. 25–40, Contemp. Math. 73, American Mathematical Society, Providence, RI, 1988
- [61] J. Dodziuk and W. S. Kendall, Combinatorial Laplacians and isoperimetric inequality. In *From local times to global geometry, control and physics (Coventry, 1984/85)*, edited by K. D. Elworthy, pp. 68–74, Pitman Res. Notes Math. Ser. 150, Longman Scientific and Technical, Harlow, 1986
- [62] J. Draisma and A. Vargas, On the gonality of metric graphs. *Notices Amer. Math. Soc.* **68** (2021), no. 5, 687–695
- [63] A. Dudko and R. Grigorchuk, On the question “Can one hear the shape of a group?” and a Hulanicki type theorem for graphs. *Israel J. Math.* **237** (2020), no. 1, 53–74
- [64] N. Enriquez and Y. Kifer, Markov chains on graphs and Brownian motion. *J. Theoret. Probab.* **14** (2001), no. 2, 495–510
- [65] A. Erschler and T. Zheng, Growth of periodic Grigorchuk groups. *Invent. Math.* **219** (2020), no. 3, 1069–1155
- [66] P. Exner, A duality between Schrödinger operators on graphs and certain Jacobi matrices. *Ann. Inst. H. Poincaré Phys. Théor.* **66** (1997), no. 4, 359–371
- [67] P. Exner, J. P. Keating, P. Kuchment, T. Sunada, and A. Teplyaev, *Analysis on graphs and its applications*. Proc. Sympos. Pure Math. 77, American Mathematical Society, Providence, RI, 2008
- [68] P. Exner, A. Kostenko, M. Malamud, and H. Neidhardt, Spectral theory of infinite quantum graphs. *Ann. Henri Poincaré* **19** (2018), no. 11, 3457–3510
- [69] P. Exner and H. Kovářík, *Quantum waveguides*. Theoret. Math. Phys., Springer, Cham, 2015
- [70] W. Feller, The parabolic differential equations and the associated semi-groups of transformations. *Ann. of Math. (2)* **55** (1952), 468–519
- [71] M. Folz, Volume growth and spectrum for general graph Laplacians. *Math. Z.* **276** (2014), no. 1–2, 115–131
- [72] M. Folz, Volume growth and stochastic completeness of graphs. *Trans. Amer. Math. Soc.* **366** (2014), no. 4, 2089–2119

- [73] L. R. Foulds, *Graph theory applications*. Universitext, Springer, New York, 1992
- [74] R. L. Frank, D. Lenz, and D. Wingert, Intrinsic metrics for non-local symmetric Dirichlet forms and applications to spectral theory. *J. Funct. Anal.* **266** (2014), no. 8, 4765–4808
- [75] R. L. Frank, E. H. Lieb, and R. Seiringer, Equivalence of Sobolev inequalities and Lieb–Thirring inequalities. In *XVIIth international congress on mathematical physics*, edited by P. Exner, pp. 523–535, World Scientific, Hackensack, NJ, 2010
- [76] H. Freudenthal, Über die Enden topologischer Räume und Gruppen. *Math. Z.* **33** (1931), no. 1, 692–713
- [77] H. Freudenthal, Über die Enden diskreter Räume und Gruppen. *Comment. Math. Helv.* **17** (1945), 1–38
- [78] M. Fukushima, Y. Oshima, and M. Takeda, *Dirichlet forms and symmetric Markov processes*. 2nd edn., De Gruyter Stud. Math. 19, Walter de Gruyter, Berlin, 2011
- [79] M. P. Gaffney, A special Stokes’s theorem for complete Riemannian manifolds. *Ann. of Math.* (2) **60** (1954), 140–145
- [80] M. P. Gaffney, Hilbert space methods in the theory of harmonic integrals. *Trans. Amer. Math. Soc.* **78** (1955), 426–444
- [81] B. Gaveau and M. Okada, Differential forms and heat diffusion on one-dimensional singular varieties. *Bull. Sci. Math.* **115** (1991), no. 1, 61–79
- [82] R. Geoghegan, *Topological methods in group theory*. Grad. Texts in Math. 243, Springer, New York, 2008
- [83] A. Georgakopoulos, S. Haeseler, M. Keller, D. Lenz, and R. K. Wojciechowski, Graphs of finite measure. *J. Math. Pures Appl.* (9) **103** (2015), no. 5, 1093–1131
- [84] I. M. Glazman, *Direct methods of qualitative spectral analysis of singular differential operators*. Daniel Davey, New York, 1965
- [85] I. C. Gohberg and M. G. Krein, *Theory and applications of Volterra operators in Hilbert space*. Transl. Math. Monogr. 24, American Mathematical Society, Providence, R.I., 1970
- [86] V. I. Gorbachuk and M. L. Gorbachuk, *Boundary value problems for operator differential equations*. Math. Appl. (Soviet Series) 48, Kluwer Academic., Dordrecht, 1991
- [87] R. I. Grigorchuk, Symmetrical random walks on discrete groups. In *Multicomponent random systems*, pp. 285–325, Adv. Probab. Related Topics 6, Dekker, New York, 1980
- [88] R. Grigorchuk, T. Nagnibeda, and A. Pérez, On spectra and spectral measures of Schreier and Cayley graphs. *Int. Math. Res. Not. IMRN* **2022** (2022), no. 15, 11957–12002
- [89] R. I. Grigorchuk and A. Żuk, The lamplighter group as a group generated by a 2-state automaton, and its spectrum. *Geom. Dedicata* **87** (2001), no. 1–3, 209–244
- [90] A. Grigor’yan, Analytic and geometric background of recurrence and non-explosion of the Brownian motion on Riemannian manifolds. *Bull. Amer. Math. Soc. (N. S.)* **36** (1999), no. 2, 135–249
- [91] A. Grigor’yan, *Introduction to analysis on graphs*. Univ. Lecture Ser. 71, American Mathematical Society, Providence, RI, 2018

- [92] A. Grigor'yan, X. Huang, and J. Masamune, On stochastic completeness of jump processes. *Math. Z.* **271** (2012), no. 3-4, 1211–1239
- [93] A. Grigor'yan and J. Masamune, Parabolicity and stochastic completeness of manifolds in terms of the Green formula. *J. Math. Pures Appl. (9)* **100** (2013), no. 5, 607–632
- [94] M. Gromov, Hyperbolic manifolds, groups and actions. In *Riemann surfaces and related topics: Proceedings of the 1978 Stony Brook Conference (State Univ. New York, Stony Brook, N.Y., 1978)*, pp. 183–213, Ann. of Math. Stud. 97, Princeton University Press, Princeton, N.J., 1981
- [95] M. Gromov, Hyperbolic groups. In *Essays in group theory*, edited by S. M. Gersten, pp. 75–263, Math. Sci. Res. Inst. Publ. 8, Springer, New York, 1987
- [96] B. Güneysu, M. Keller, and M. Schmidt, A Feynman–Kac–Itô formula for magnetic Schrödinger operators on graphs. *Probab. Theory Related Fields* **165** (2016), no. 1-2, 365–399
- [97] S. Haeseler, Analysis of Dirichlet forms on graphs. PhD thesis, University of Jena, 2014, arXiv:[1705.06322](https://arxiv.org/abs/1705.06322)
- [98] S. Haeseler, M. Keller, D. Lenz, J. Masamune, and M. Schmidt, Global properties of Dirichlet forms in terms of Green's formula. *Calc. Var. Partial Differential Equations* **56** (2017), no. 5, art. 124
- [99] S. Haeseler, M. Keller, D. Lenz, and R. Wojciechowski, Laplacians on infinite graphs: Dirichlet and Neumann boundary conditions. *J. Spectr. Theory* **2** (2012), no. 4, 397–432
- [100] S. Haeseler, M. Keller, and R. K. Wojciechowski, Volume growth and bounds for the essential spectrum for Dirichlet forms. *J. Lond. Math. Soc. (2)* **88** (2013), no. 3, 883–898
- [101] O. Häggström, J. Jonasson, and R. Lyons, Explicit isoperimetric constants and phase transitions in the random-cluster model. *Ann. Probab.* **30** (2002), no. 1, 443–473
- [102] R. Halin, Über unendliche Wege in Graphen. *Math. Ann.* **157** (1964), 125–137
- [103] B. M. Hambly and T. Kumagai, Heat kernel estimates for symmetric random walks on a class of fractal graphs and stability under rough isometries. In *Fractal geometry and applications: A jubilee of Benoît Mandelbrot, Part 2*, edited by M. L. Lapidus, pp. 233–259, Proc. Sympos. Pure Math. 72, American Mathematical Society, Providence, RI, 2004
- [104] Y. Higuchi, Combinatorial curvature for planar graphs. *J. Graph Theory* **38** (2001), no. 4, 220–229
- [105] Y. Higuchi and T. Shirai, Isoperimetric constants of  $(d, f)$ -regular planar graphs. *Interdiscip. Inform. Sci.* **9** (2003), no. 2, 221–228
- [106] I. Holopainen, Rough isometries and  $p$ -harmonic functions with finite Dirichlet integral. *Rev. Mat. Iberoam.* **10** (1994), no. 1, 143–176
- [107] I. Holopainen and P. M. Soardi,  $p$ -harmonic functions on graphs and manifolds. *Manuscripta Math.* **94** (1997), no. 1, 95–110
- [108] I. Holopainen and P. M. Soardi, A strong Liouville theorem for  $p$ -harmonic functions on graphs. *Ann. Acad. Sci. Fenn. Math.* **22** (1997), no. 1, 205–226

- [109] H. Hopf, Enden offener Räume und unendliche diskontinuierliche Gruppen. *Comment. Math. Helv.* **16** (1944), 81–100
- [110] B. Hua and J. Jost,  $L^q$  harmonic functions on graphs. *Israel J. Math.* **202** (2014), no. 1, 475–490
- [111] B. Hua and J. Jost, Geometric analysis aspects of infinite semiplanar graphs with non-negative curvature II. *Trans. Amer. Math. Soc.* **367** (2015), no. 4, 2509–2526
- [112] B. Hua, J. Jost, and S. Liu, Geometric analysis aspects of infinite semiplanar graphs with nonnegative curvature. *J. Reine Angew. Math.* **700** (2015), 1–36
- [113] B. Hua and M. Keller, Harmonic functions of general graph Laplacians. *Calc. Var. Partial Differential Equations* **51** (2014), no. 1-2, 343–362
- [114] X. Huang, A note on the volume growth criterion for stochastic completeness of weighted graphs. *Potential Anal.* **40** (2014), no. 2, 117–142
- [115] X. Huang, M. Keller, J. Masamune, and R. K. Wojciechowski, A note on self-adjoint extensions of the Laplacian on weighted graphs. *J. Funct. Anal.* **265** (2013), no. 8, 1556–1578
- [116] X. Huang, M. Keller, and M. Schmidt, On the uniqueness class, stochastic completeness and volume growth for graphs. *Trans. Amer. Math. Soc.* **373** (2020), no. 12, 8861–8884
- [117] X. Huang and Y. Shiozawa, Upper escape rate of Markov chains on weighted graphs. *Stochastic Process. Appl.* **124** (2014), no. 1, 317–347
- [118] M. Ishida, *Pseudo-curvature of a graph*. Lecture at “Workshop on topological graph theory”, Yokohama National University, Yokohama, 1990.
- [119] I. S. Kac and M. G. Kreĭn, Criteria for the discreteness of the spectrum of a singular string. *Izv. Vysš. Učebn. Zaved. Matematika* **1958** (1958), no. 2 (3), 136–153
- [120] I. S. Kac and M. G. Krein, On the spectral functions of the string. *Amer. Math. Soc. Transl. Ser. 2* **103** (1974), 19–102
- [121] S. Kakutani, Random walk and the type problem of Riemann surfaces. In *Contributions to the theory of Riemann surfaces*, edited by L. Ahlfors, pp. 95–101, Ann. of Math. Stud. 30, Princeton University Press, Princeton, N.J., 1953
- [122] M. Kanai, Rough isometries, and combinatorial approximations of geometries of non-compact Riemannian manifolds. *J. Math. Soc. Japan* **37** (1985), no. 3, 391–413
- [123] M. Kanai, Rough isometries and the parabolicity of Riemannian manifolds. *J. Math. Soc. Japan* **38** (1986), no. 2, 227–238
- [124] L. Karp, Subharmonic functions on real and complex manifolds. *Math. Z.* **179** (1982), no. 4, 535–554
- [125] A. Kasue, Convergence of metric graphs and energy forms. *Rev. Mat. Iberoam.* **26** (2010), no. 2, 367–448
- [126] T. Kato, *Perturbation theory for linear operators*. 2nd edn., Grundlehren Math. Wiss. 132, Springer, Berlin, 1976
- [127] I. S. Kats, The spectral theory of a string. *Ukrainian Math. J.* **46** (1994), no. 3, 159–182

- [128] M. Keller, Curvature, geometry and spectral properties of planar graphs. *Discrete Comput. Geom.* **46** (2011), no. 3, 500–525
- [129] M. Keller, Intrinsic metrics on graphs: A survey. In *Mathematical technology of networks*, edited by D. Mugnolo, pp. 81–119, Springer Proc. Math. Stat. 128, Springer, Cham, 2015
- [130] M. Keller, Geometric and spectral consequences of curvature bounds on tessellations. In *Modern approaches to discrete curvature*, edited by L. Najman and P. Romon, pp. 175–209, Lecture Notes in Math. 2184, Springer, Cham, 2017
- [131] M. Keller and D. Lenz, Unbounded Laplacians on graphs: Basic spectral properties and the heat equation. *Math. Model. Nat. Phenom.* **5** (2010), no. 4, 198–224
- [132] M. Keller and D. Lenz, Dirichlet forms and stochastic completeness of graphs and subgraphs. *J. Reine Angew. Math.* **666** (2012), 189–223
- [133] M. Keller, D. Lenz, M. Schmidt, and M. Schwarz, Boundary representation of Dirichlet forms on discrete spaces. *J. Math. Pures Appl. (9)* **126** (2019), 109–143
- [134] M. Keller, D. Lenz, M. Schmidt, and R. K. Wojciechowski, Note on uniformly transient graphs. *Rev. Mat. Iberoam.* **33** (2017), no. 3, 831–860
- [135] M. Keller, D. Lenz, and R. K. Wojciechowski, Volume growth, spectrum and stochastic completeness of infinite graphs. *Math. Z.* **274** (2013), no. 3-4, 905–932
- [136] M. Keller, D. Lenz, and R. K. Wojciechowski, *Graphs and discrete Dirichlet spaces*. Grundlehren Math. Wiss. 358, Springer, Cham, 2021
- [137] M. Keller and F. Münch, A new discrete Hopf–Rinow theorem. *Discrete Math.* **342** (2019), no. 9, 2751–2757
- [138] M. Keller and N. Peyerimhoff, Cheeger constants, growth and spectrum of locally tessellating planar graphs. *Math. Z.* **268** (2011), no. 3–4, 871–886
- [139] M. Keller, Y. Pinchover, and F. Pogorzelski, Optimal Hardy inequalities for Schrödinger operators on graphs. *Comm. Math. Phys.* **358** (2018), no. 2, 767–790
- [140] M. Keller, Y. Pinchover, and F. Pogorzelski, Criticality theory for Schrödinger operators on graphs. *J. Spectr. Theory* **10** (2020), no. 1, 73–114
- [141] H. Kesten, Full Banach mean values on countable groups. *Math. Scand.* **7** (1959), 146–156
- [142] H. Kesten, Symmetric random walks on groups. *Trans. Amer. Math. Soc.* **92** (1959), 336–354
- [143] A. S. Kostenko and M. M. Malamud, 1-D Schrödinger operators with local point interactions on a discrete set. *J. Differential Equations* **249** (2010), no. 2, 253–304
- [144] A. Kostenko and M. Malamud, 1-D Schrödinger operators with local point interactions: A review. In *Spectral analysis, differential equations and mathematical physics: A festschrift in honor of Fritz Gesztesy's 60th birthday*, edited by H. Holden, pp. 235–262, Proc. Sympos. Pure Math. 87, American Mathematical Society, Providence, RI, 2013
- [145] A. Kostenko, M. Malamud, and N. Nicolussi, A Glazman–Povzner–Wienholtz theorem on graphs. *Adv. Math.* **395** (2022), art. 108158, 30

- [146] A. Kostenko, D. Mugnolo, and N. Nicolussi, Self-adjoint and Markovian extensions of infinite quantum graphs. *J. Lond. Math. Soc.* (2) **105** (2022), no. 2, 1262–1313
- [147] A. Kostenko and N. Nicolussi, Spectral estimates for infinite quantum graphs. *Calc. Var. Partial Differential Equations* **58** (2019), no. 1, art. 15
- [148] A. Kostenko and N. Nicolussi, A note on the Gaffney Laplacian on infinite metric graphs. *J. Funct. Anal.* **281** (2021), no. 10, art. 109216
- [149] A. Kostenko and N. Nicolussi, Quantum graphs on radially symmetric antitrees. *J. Spectr. Theory* **11** (2021), no. 2, 411–460
- [150] V. Kostrykin and R. Schrader, Kirchhoff’s rule for quantum wires. *J. Phys. A* **32** (1999), no. 4, 595–630
- [151] Y. H. Lee, Rough isometry and Dirichlet finite harmonic functions on Riemannian manifolds. *Manuscripta Math.* **99** (1999), no. 3, 311–328
- [152] D. Levin and M. Solomyak, The Rozenblum–Lieb–Cwikel inequality for Markov generators. *J. Anal. Math.* **71** (1997), 173–193
- [153] P. Li and R. Schoen,  $L^p$  and mean value properties of subharmonic functions on Riemannian manifolds. *Acta Math.* **153** (1984), no. 3-4, 279–301
- [154] T. Lupu, From loop clusters and random interlacements to the free field. *Ann. Probab.* **44** (2016), no. 3, 2117–2146
- [155] T. Lyons, Instability of the Liouville property for quasi-isometric Riemannian manifolds and reversible Markov chains. *J. Differential Geom.* **26** (1987), no. 1, 33–66
- [156] M. Malamud and H. Neidhardt, Sturm–Liouville boundary value problems with operator potentials and unitary equivalence. *J. Differential Equations* **252** (2012), no. 11, 5875–5922
- [157] M. M. Malamud, On a formula for the generalized resolvents of a non-densely defined Hermitian operator. *Ukrainian Math. J.* **44** (1992), no. 12, 1522–1547
- [158] M. M. Malamud, Some classes of extensions of a Hermitian operator with lacunae. *Ukrainian Math. J.* **44** (1992), no. 2, 190–204
- [159] A. Mann, *How groups grow*. London Math. Soc. Lecture Note Ser. 395, Cambridge University Press, Cambridge, 2012
- [160] S. Markvorsen, S. McGuinness, and C. Thomassen, Transient random walks on graphs and metric spaces with applications to hyperbolic surfaces. *Proc. Lond. Math. Soc.* (3) **64** (1992), no. 1, 1–20
- [161] J. T. Martí, Evaluation of the least constant in Sobolev’s inequality for  $H^1(0, s)$ . *SIAM J. Numer. Anal.* **20** (1983), no. 6, 1239–1242
- [162] J. Masamune, Essential self-adjointness of Laplacians on Riemannian manifolds with fractal boundary. *Comm. Partial Differential Equations* **24** (1999), no. 3-4, 749–757
- [163] J. Masamune, Analysis of the Laplacian of an incomplete manifold with almost polar boundary. *Rend. Mat. Appl.* (7) **25** (2005), no. 1, 109–126
- [164] J. Masamune, A Liouville property and its application to the Laplacian of an infinite graph. In *Spectral analysis in geometry and number theory*, edited by M. Kotani, pp. 103–115, Contemp. Math. 484, American Mathematical Society, Providence, RI, 2009

- [165] J. Masamune and T. Uemura, Conservation property of symmetric jump processes. *Ann. Inst. Henri Poincaré Probab. Stat.* **47** (2011), no. 3, 650–662
- [166] H. P. McKean, An upper bound to the spectrum of  $\Delta$  on a manifold of negative curvature. *J. Differential Geom.* **4** (1970), 359–366
- [167] O. Milatovic, Essential self-adjointness of magnetic Schrödinger operators on locally finite graphs. *Integral Equations Operator Theory* **71** (2011), no. 1, 13–27
- [168] A. Murakami and M. Yamasaki, An introduction of Kuramochi boundary of an infinite network. *Mem. Fac. Sci. Eng. Shimane Univ. Ser. B Math. Sci.* **30** (1997), 57–89
- [169] K. Naimark and M. Solomyak, Eigenvalue estimates for the weighted Laplacian on metric trees. *Proc. Lond. Math. Soc. (3)* **80** (2000), no. 3, 690–724
- [170] K. Naimark and M. Solomyak, Geometry of Sobolev spaces on regular trees and the Hardy inequalities. *Russ. J. Math. Phys.* **8** (2001), no. 3, 322–335
- [171] S. Nicaise, Some results on spectral theory over networks, applied to nerve impulse transmission. In *Orthogonal polynomials and applications (Bar-le-Duc, 1984)*, pp. 532–541, Lecture Notes in Math. 1171, Springer, Berlin, 1985
- [172] S. Nicaise, Spectre des réseaux topologiques finis. *Bull. Sci. Math. (2)* **111** (1987), no. 4, 401–413
- [173] N. Nicolussi, Strong isoperimetric inequality for tessellating quantum graphs. In *Discrete and continuous models in the theory of networks*, edited by F. Fatihcan, pp. 271–290, Oper. Theory Adv. Appl. 281, Birkhäuser/Springer, Cham, [2020] ©2020
- [174] J. R. Norris, *Markov chains*. Camb. Ser. Stat. Probab. Math. 2, Cambridge University Press, Cambridge, 1998
- [175] P. W. Nowak and G. Yu, *Large scale geometry*. EMS Textbk. Math., European Mathematical Society (EMS), Zürich, 2012
- [176] B.-G. Oh, Duality properties of strong isoperimetric inequalities on a planar graph and combinatorial curvatures. *Discrete Comput. Geom.* **51** (2014), no. 4, 859–884
- [177] B.-G. Oh, Sharp isoperimetric inequalities for infinite plane graphs with bounded vertex and face degrees. 2020, arXiv:[2009.04394](https://arxiv.org/abs/2009.04394)
- [178] A. Yu. Ol'shanskii, On the question of the existence of an invariant mean on a group. *Russian Math. Surveys* **35** (1980), no. 4, 180–181
- [179] K. Pankrashkin, Unitary dimension reduction for a class of self-adjoint extensions with applications to graph-like structures. *J. Math. Anal. Appl.* **396** (2012), no. 2, 640–655
- [180] M. Plümer, Upper eigenvalue bounds for the Kirchhoff Laplacian on embedded metric graphs. *J. Spectr. Theory* **11** (2021), no. 4, 1857–1894
- [181] O. Post, First order approach and index theorems for discrete and metric graphs. *Ann. Henri Poincaré* **10** (2009), no. 5, 823–866
- [182] O. Post, *Spectral analysis on graph-like spaces*. Lecture Notes in Math. 2039, Springer, Heidelberg, 2012
- [183] A. Y. Povzner, On the expansion of arbitrary functions in characteristic functions of the operator  $-\Delta u + cu$ . *Mat. Sbornik N.S.* **32(74)** (1953), 109–156

- [184] M. Reed and B. Simon, *Methods of modern mathematical physics. II. Fourier analysis, self-adjointness*. Academic Press, New York, 1975
- [185] M. Reed and B. Simon, *Methods of modern mathematical physics, I: Functional analysis*. Revised and enlarged edn., Academic Press, New York, 1980
- [186] M. Rigoli, M. Salvatori, and M. Vignati, Subharmonic functions on graphs. *Israel J. Math.* **99** (1997), 1–27
- [187] J. Roe, *Lectures on coarse geometry*. Univ. Lecture Ser. 31, American Mathematical Society, Providence, RI, 2003
- [188] F. S. Rofe-Beketov, Selfadjoint extensions of differential operators in a space of vector-valued functions (in Russian). *Teor. Funkcií Funkcional. Anal. i Prilozhen. Vyp. 8* (1969), 3–24
- [189] G. V. Rozenblyum and M. Z. Solomyak, On spectral estimates for Schrödinger-type operators: The case of small local dimension. *Funct. Anal. Appl.* **44** (2010), no. 4, 259–269
- [190] M. Schmidt, On the existence and uniqueness of self-adjoint realizations of discrete (magnetic) Schrödinger operators. In *Analysis and geometry on graphs and manifolds*, edited by M. Keller, pp. 250–327, London Math. Soc. Lecture Note Ser. 461, Cambridge University Press, Cambridge, 2020
- [191] K. Schmüdgen, *Unbounded self-adjoint operators on Hilbert space*. Grad. Texts in Math. 265, Springer, Dordrecht, 2012
- [192] M. A. Shubin, Spectral theory of elliptic operators on noncompact manifolds. In *Méthodes semi-classiques. Vol. 1. École d’été (Nantes, 1991)*, edited by D. Robert, pp. 35–108, Astérisque 207, Société Mathématique de France, Paris, 1992
- [193] P. M. Soardi, Recurrence and transience of the edge graph of a tiling of the Euclidean plane. *Math. Ann.* **287** (1990), no. 4, 613–626
- [194] P. M. Soardi, Rough isometries and Dirichlet finite harmonic functions on graphs. *Proc. Amer. Math. Soc.* **119** (1993), no. 4, 1239–1248
- [195] P. M. Soardi, *Potential theory on infinite networks*. Lecture Notes in Math. 1590, Springer, Berlin, 1994
- [196] M. Solomyak, On the spectrum of the Laplacian on regular metric trees. *Waves Random Media* **14** (2004), S155–S171
- [197] D. A. Stone, A combinatorial analogue of a theorem of Myers. *Illinois J. Math.* **20** (1976), no. 1, 12–21
- [198] K.-T. Sturm, Analysis on local Dirichlet spaces. I. Recurrence, conservativeness and  $L^p$ -Liouville properties. *J. Reine Angew. Math.* **456** (1994), 173–196
- [199] K.-T. Sturm, Analysis on local Dirichlet spaces. II. Upper Gaussian estimates for the fundamental solutions of parabolic equations. *Osaka J. Math.* **32** (1995), no. 2, 275–312
- [200] K. T. Sturm, Analysis on local Dirichlet spaces. III. The parabolic Harnack inequality. *J. Math. Pures Appl. (9)* **75** (1996), no. 3, 273–297

- [201] P. W. Sy and T. Sunada, Discrete Schrödinger operators on a graph. *Nagoya Math. J.* **125** (1992), 141–150
- [202] N. Torki-Hamza, *Laplaciens de graphes infinis, I – Graphes métriquement complets*. *Confluentes Math.* **2** (2010), no. 3, 333–350; English transl. in arXiv:[1201.4644](https://arxiv.org/abs/1201.4644)
- [203] N. T. Varopoulos, Brownian motion and transient groups. *Ann. Inst. Fourier (Grenoble)* **33** (1983), no. 2, 241–261
- [204] N. T. Varopoulos, Hardy–Littlewood theory for semigroups. *J. Funct. Anal.* **63** (1985), no. 2, 240–260
- [205] N. T. Varopoulos, Long range estimates for Markov chains. *Bull. Sci. Math. (2)* **109** (1985), no. 3, 225–252
- [206] N. T. Varopoulos, L. Saloff-Coste, and T. Coulhon, *Analysis and geometry on groups*. Cambridge Tracts in Math. 100, Cambridge University Press, Cambridge, 1992
- [207] J. von Below, A characteristic equation associated to an eigenvalue problem on  $c^2$ -networks. *Linear Algebra Appl.* **71** (1985), 309–325
- [208] J. Weidmann, *Spectral theory of ordinary differential operators*. Lecture Notes in Math. 1258, Springer, Berlin, 1987
- [209] H. Weyl, Über gewöhnliche Differentialgleichungen mit Singularitäten und die zugehörigen Entwicklungen willkürlicher Funktionen. *Math. Ann.* **68** (1910), no. 2, 220–269
- [210] E. Wienholtz, Halbbeschränkte partielle Differentialoperatoren zweiter Ordnung vom elliptischen Typus. *Math. Ann.* **135** (1958), 50–80
- [211] W. Woess, A note on tilings and strong isoperimetric inequality. *Math. Proc. Cambridge Philos. Soc.* **124** (1998), no. 3, 385–393
- [212] W. Woess, *Random walks on infinite graphs and groups*. Cambridge Tracts in Math. 138, Cambridge University Press, Cambridge, 2000
- [213] R. K. Wojciechowski, Stochastically incomplete manifolds and graphs. In *Random walks, boundaries and spectra*, edited by D. Lenz, pp. 163–179, Progr. Probab. 64, Birkhäuser/Springer, Basel, 2011
- [214] R. K. Wojciechowski, Stochastic completeness of graphs: Bounded Laplacians, intrinsic metrics, volume growth and curvature. *J. Fourier Anal. Appl.* **27** (2021), no. 2, art. 30
- [215] A. Wouk, Difference equations and  $J$ -matrices. *Duke Math. J.* **20** (1953), 141–159
- [216] J. Wysoczański, Royden compactification of integers. *Hiroshima Math. J.* **26** (1996), no. 3, 515–529
- [217] S. T. Yau, Some function-theoretic properties of complete Riemannian manifold and their applications to geometry. *Indiana Univ. Math. J.* **25** (1976), no. 7, 659–670
- [218] A. Żuk, On the norms of the random walks on planar graphs. *Ann. Inst. Fourier (Grenoble)* **47** (1997), no. 5, 1463–1490