

References

- [1] M. Abramowitz and I. A. Stegun (eds.), *Handbook of mathematical functions*. Dover Publications, New York, 1972
- [2] C. Adam, B. Muratori, and C. Nash, [Zero modes of the Dirac operator in three dimensions](#). *Phys. Rev. D (3)* **60** (1999), no. 12, Paper No. 125001
- [3] C. Adam, B. Muratori, and C. Nash, [Degeneracy of zero modes of the Dirac operator in three dimensions](#). *Phys. Lett. B* **485** (2000), no. 1-3, 314–318
- [4] C. Adam, B. Muratori, and C. Nash, [Multiple zero modes of the Dirac operator in three dimensions](#). *Phys. Rev. D (3)* **62** (2000), no. 8, Paper No. 085026
- [5] C. Adam, B. Muratori, and C. Nash, [Zero modes in finite range magnetic fields](#). *Modern Phys. Lett. A* **15** (2000), no. 25, 1577–1581
- [6] S. Agmon, Spectral properties of Schrödinger operators and scattering theory. *Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4)* **2** (1975), no. 2, 151–218
- [7] Y. Aharonov and A. Casher, [Ground state of a spin- \$\frac{1}{2}\$ charged particle in a two-dimensional magnetic field](#). *Phys. Rev. A (3)* **19** (1979), no. 6, 2461–2462
- [8] D. Aiba, [Absence of zero resonances of massless Dirac operators](#). *Hokkaido Math. J.* **45** (2016), no. 2, 263–270
- [9] S. Albeverio, [On bound states in the continuum of \$N\$ -body systems and the virial theorem](#). *Ann. Physics* **71** (1972), 167–276
- [10] S. Albeverio, F. Gesztesy, and R. Høegh-Krohn, The low energy expansion in nonrelativistic scattering theory. *Ann. Inst. H. Poincaré Sect. A (N.S.)* **37** (1982), no. 1, 1–28
- [11] W. O. Amrein, A. Boutet de Monvel, and V. Georgescu, *Co-groups, commutator methods and spectral theory of N -body Hamiltonians*. Progr. Math. 135, Birkhäuser, Basel, 1996
- [12] R. B. Ash and W. P. Novinger, *Complex variables*. 2nd edn., Dover Publications, New York, 2007
- [13] A. A. Balinsky and W. D. Evans, [On the zero modes of Pauli operators](#). *J. Funct. Anal.* **179** (2001), no. 1, 120–135
- [14] A. A. Balinsky and W. D. Evans, [On the zero modes of Weyl–Dirac operators and their multiplicity](#). *Bull. London Math. Soc.* **34** (2002), no. 2, 236–242
- [15] A. A. Balinsky and W. D. Evans, [Zero modes of Pauli and Weyl–Dirac operators](#). In *Advances in differential equations and mathematical physics (Birmingham, AL, 2002)*, pp. 1–9, Contemp. Math. 327, American Mathematical Society, Providence, RI, 2003
- [16] A. A. Balinsky and W. D. Evans, *Spectral analysis of relativistic operators*. Imperial College Press, London, 2011
- [17] A. A. Balinsky, W. D. Evans, and Y. Saitō, [Dirac–Sobolev inequalities and estimates for the zero modes of massless Dirac operators](#). *J. Math. Phys.* **49** (2008), no. 4, Paper No. 043514
- [18] E. Balslev and B. Helffer, [Limiting absorption principle and resonances for the Dirac operator](#). *Adv. in Appl. Math.* **13** (1992), no. 2, 186–215

- [19] H. Baumgärtel and M. Wollenberg, *Mathematical scattering theory*. Mathematische Lehrbücher und Monographien, II. Abteilung: Mathematische Monographien 59, Akademie, Berlin, 1983
- [20] J. Behrndt, F. Gesztesy, H. Holden, and R. Nichols, *Dirichlet-to-Neumann maps, abstract Weyl–Titchmarsh M -functions, and a generalized index of unbounded meromorphic operator-valued functions*. *J. Differential Equations* **261** (2016), no. 6, 3551–3587
- [21] M. Ben-Artzi and A. Devinatz, *The limiting absorption principle for partial differential operators*. *Mem. Amer. Math. Soc.* **66** (1987), no. 364, iv+70
- [22] M. Ben-Artzi and J. Nemirovsky, Remarks on relativistic Schrödinger operators and their extensions. *Ann. Inst. H. Poincaré Phys. Théor.* **67** (1997), no. 1, 29–39
- [23] M. Ben-Artzi and T. Umeda, *Spectral theory of first-order systems: From crystals to Dirac operators*. *Rev. Math. Phys.* **33** (2021), no. 5, Paper No. 2150014, 52 pp.
- [24] R. D. Benguria and H. Van Den Bosch, *A criterion for the existence of zero modes for the Pauli operator with fastly decaying fields*. *J. Math. Phys.* **56** (2015), no. 5, Paper No. 052104
- [25] A. Berthier and V. Georgescu, *On the point spectrum of Dirac operators*. *J. Funct. Anal.* **71** (1987), no. 2, 309–338
- [26] M. S. Birman, G. E. Karadzhov, and M. Z. Solomyak, Boundedness conditions and spectrum estimates for the operators $b(X)a(D)$ and their analogs. In *Estimates and asymptotics for discrete spectra of integral and differential equations (Leningrad, 1989–90)*, pp. 85–106, Adv. Soviet Math. 7, American Mathematical Society, Providence, RI, 1991
- [27] M. S. Birman and M. Z. Solomjak, *Spectral theory of selfadjoint operators in Hilbert space*. Math. Appl. (Soviet Ser.), D. Reidel Publishing, Dordrecht, 1987
- [28] H. Blancarte, B. Grébert, and R. Weder, *High- and low-energy estimates for the Dirac equation*. *J. Math. Phys.* **36** (1995), no. 3, 991–1015
- [29] D. Bollé, F. Gesztesy, and C. Danneels, Threshold scattering in two dimensions. *Ann. Inst. H. Poincaré Phys. Théor.* **48** (1988), no. 2, 175–204
- [30] D. Bollé, F. Gesztesy, C. Danneels, and S. F. J. Wilk, *Threshold behavior and Levinson’s theorem for two-dimensional scattering systems: A surprise*. *Phys. Rev. Lett.* **56** (1986), no. 9, 900–903
- [31] D. Bollé, F. Gesztesy, H. Grosse, W. Schweiger, and B. Simon, *Witten index, axial anomaly, and Krein’s spectral shift function in supersymmetric quantum mechanics*. *J. Math. Phys.* **28** (1987), no. 7, 1512–1525
- [32] N. Boussaid and A. Comech, Virtual levels and virtual states of linear operators in Banach spaces. Applications to Schrödinger operators. 2021, arXiv:[2101.11979](https://arxiv.org/abs/2101.11979)
- [33] N. Boussaid and S. Golénia, *Limiting absorption principle for some long range perturbations of Dirac systems at threshold energies*. *Comm. Math. Phys.* **299** (2010), no. 3, 677–708
- [34] A. Boutet de Monvel and M. Mantoiu, *The method of the weakly conjugate operator*. In *Inverse and algebraic quantum scattering theory (Lake Balaton, 1996)*, pp. 204–226, Lecture Notes in Phys. 488, Springer, Berlin, 1997

- [35] A. Boutet de Monvel-Berthier, D. Manda, and R. Purice, Limiting absorption principle for the Dirac operator. *Ann. Inst. H. Poincaré Phys. Théor.* **58** (1993), no. 4, 413–431
- [36] T. Britz, A. Carey, F. Gesztesy, R. Nichols, F. Sukochev, and D. Zanin, [The product formula for regularized Fredholm determinants](#). *Proc. Amer. Math. Soc. Ser. B* **8** (2021), 42–51
- [37] A.-P. Calderón, [Intermediate spaces and interpolation, the complex method](#). *Studia Math.* **24** (1964), 113–190
- [38] A. Carey, F. Gesztesy, H. Grosse, G. Levitina, D. Potapov, F. Sukochev, and D. Zanin, [Trace formulas for a class of non-Fredholm operators: A review](#). *Rev. Math. Phys.* **28** (2016), no. 10, Paper No. 1630002, 55 pp.
- [39] A. Carey, F. Gesztesy, J. Kaad, G. Levitina, R. Nichols, D. Potapov, and F. Sukochev, [On the global limiting absorption principle for massless Dirac operators](#). *Ann. Henri Poincaré* **19** (2018), no. 7, 1993–2019
- [40] A. Carey, F. Gesztesy, G. Levitina, R. Nichols, D. Potapov, and F. Sukochev, [Double operator integral methods applied to continuity of spectral shift functions](#). *J. Spectr. Theory* **6** (2016), no. 4, 747–779
- [41] A. Carey, F. Gesztesy, G. Levitina, D. Potapov, F. Sukochev, and D. Zanin, [On index theory for non-Fredholm operators: a \$\(1+1\)\$ -dimensional example](#). *Math. Nachr.* **289** (2016), no. 5-6, 575–609
- [42] A. Carey, F. Gesztesy, G. Levitina, and F. Sukochev, [On the index of a non-Fredholm model operator](#). *Oper. Matrices* **10** (2016), no. 4, 881–914
- [43] A. Carey, F. Gesztesy, D. Potapov, F. Sukochev, and Y. Tomilov, [On the Witten index in terms of spectral shift functions](#). *J. Anal. Math.* **132** (2017), 1–61
- [44] A. Carey, G. Levitina, D. Potapov, and F. Sukochev, [The Witten index and the spectral shift function](#). *Rev. Math. Phys.* **34** (2022), no. 5, Paper No. 2250011, 44 pp.
- [45] R. W. Carey and J. D. Pincus, [An invariant for certain operator algebras](#). *Proc. Nat. Acad. Sci. U.S.A.* **71** (1974), 1952–1956
- [46] H. O. Cordes, [A precise pseudodifferential Foldy–Wouthuysen transform for the Dirac equation](#). *J. Evol. Equ.* **4** (2004), no. 1, 125–138
- [47] T. Daudé, [Scattering theory for massless Dirac fields with long-range potentials](#). *J. Math. Pures Appl. (9)* **84** (2005), no. 5, 615–665
- [48] L. De Carli and T. Okaji, [Strong unique continuation property for the Dirac equation](#). *Publ. Res. Inst. Math. Sci.* **35** (1999), no. 6, 825–846
- [49] S. A. Denisov, [On the absolutely continuous spectrum of Dirac operator](#). *Comm. Partial Differential Equations* **29** (2004), no. 9-10, 1403–1428
- [50] J. Dereziński and V. Jakšić, [Spectral theory of Pauli–Fierz operators](#). *J. Funct. Anal.* **180** (2001), no. 2, 243–327
- [51] P. G. Dodds and D. H. Fremlin, [Compact operators in Banach lattices](#). *Israel J. Math.* **34** (1979), no. 4, 287–320 (1980)
- [52] C. A. Downing and M. E. Portnoi, [Massless Dirac fermions in two dimensions: Confinement in nonuniform magnetic fields](#). *Phys. Rev. B* **94** (2016), Paper No. 165407

- [53] N. du Plessis, *An introduction to potential theory*. Univ. Math. Monogr. 7, Oliver and Boyd, Edinburgh, 1970
- [54] K.-J. Eckardt, Scattering theory for Dirac operators. *Math. Z.* **139** (1974), 105–131
- [55] D. M. Elton, New examples of zero modes. *J. Phys. A* **33** (2000), no. 41, 7297–7303
- [56] D. M. Elton, Spectral properties of the equation $(\nabla + ieA) \times u = \pm mu$. *Proc. Roy. Soc. Edinburgh Sect. A* **131** (2001), no. 5, 1065–1089
- [57] D. M. Elton, The local structure of zero mode producing magnetic potentials. *Comm. Math. Phys.* **229** (2002), no. 1, 121–139
- [58] M. B. Erdogan, M. Goldberg, and W. R. Green, Dispersive estimates for four dimensional Schrödinger and wave equations with obstructions at zero energy. *Comm. Partial Differential Equations* **39** (2014), no. 10, 1936–1964
- [59] M. B. Erdogan, M. Goldberg, and W. R. Green, Limiting absorption principle and Strichartz estimates for Dirac operators in two and higher dimensions. *Comm. Math. Phys.* **367** (2019), no. 1, 241–263
- [60] M. B. Erdogan, M. Goldberg, and W. R. Green, The massless Dirac equation in two dimensions: zero-energy obstructions and dispersive estimates. *J. Spectr. Theory* **11** (2021), no. 3, 935–979
- [61] M. B. Erdogan, M. Goldberg, and W. Schlag, Strichartz and smoothing estimates for Schrödinger operators with large magnetic potentials in \mathbb{R}^3 . *J. Eur. Math. Soc. (JEMS)* **10** (2008), no. 2, 507–531
- [62] M. B. Erdogan, M. Goldberg, and W. Schlag, Strichartz and smoothing estimates for Schrödinger operators with almost critical magnetic potentials in three and higher dimensions. *Forum Math.* **21** (2009), no. 4, 687–722
- [63] M. B. Erdogan and W. R. Green, Dispersive estimates for the Schrödinger equation for $C^{\frac{n-3}{2}}$ potentials in odd dimensions. *Int. Math. Res. Not. IMRN* **2010** (2010), no. 13, 2532–2565
- [64] M. B. Erdogan and W. R. Green, Dispersive estimates for Schrödinger operators in dimension two with obstructions at zero energy. *Trans. Amer. Math. Soc.* **365** (2013), no. 12, 6403–6440
- [65] M. B. Erdogan and W. R. Green, The Dirac equation in two dimensions: dispersive estimates and classification of threshold obstructions. *Comm. Math. Phys.* **352** (2017), no. 2, 719–757
- [66] M. B. Erdogan, W. R. Green, and E. Toprak, Dispersive estimates for massive Dirac operators in dimension two. *J. Differential Equations* **264** (2018), no. 9, 5802–5837
- [67] M. B. Erdogan, W. R. Green, and E. Toprak, Dispersive estimates for Dirac operators in dimension three with obstructions at threshold energies. *Amer. J. Math.* **141** (2019), no. 5, 1217–1258
- [68] M. B. Erdogan and W. Schlag, Dispersive estimates for Schrödinger operators in the presence of a resonance and/or an eigenvalue at zero energy in dimension three. I. *Dyn. Partial Differ. Equ.* **1** (2004), no. 4, 359–379

- [69] M. B. Erdogan and W. Schlag, [Dispersive estimates for Schrödinger operators in the presence of a resonance and/or an eigenvalue at zero energy in dimension three. II.](#) *J. Anal. Math.* **99** (2006), 199–248
- [70] L. Erdős and J. P. Solovej, [The kernel of Dirac operators on \$\mathbb{S}^3\$ and \$\mathbb{R}^3\$.](#) *Rev. Math. Phys.* **13** (2001), no. 10, 1247–1280
- [71] R. L. Frank, [Eigenvalue bounds for Schrödinger operators with complex potentials. III.](#) *Trans. Amer. Math. Soc.* **370** (2018), no. 1, 219–240
- [72] R. L. Frank and M. Loss, [A sharp criterion for zero modes of the Dirac equation.](#) 2022, arXiv:[2201.03610](https://arxiv.org/abs/2201.03610), to appear in *J. Eur. Math. Soc. (JEMS)*
- [73] J. Fröhlich, E. H. Lieb, and M. Loss, [Stability of Coulomb systems with magnetic fields. I. The one-electron atom.](#) *Comm. Math. Phys.* **104** (1986), no. 2, 251–270
- [74] V. Georgescu and M. Măntoiu, [On the spectral theory of singular Dirac type Hamiltonians.](#) *J. Operator Theory* **46** (2001), no. 2, 289–321
- [75] C. Gérard, [A proof of the abstract limiting absorption principle by energy estimates.](#) *J. Funct. Anal.* **254** (2008), no. 11, 2707–2724
- [76] F. Gesztesy and H. Holden, [A unified approach to eigenvalues and resonances of Schrödinger operators using Fredholm determinants.](#) *J. Math. Anal. Appl.* **123** (1987), no. 1, 181–198
- [77] F. Gesztesy, H. Holden, and R. Nichols, [On factorizations of analytic operator-valued functions and eigenvalue multiplicity questions.](#) *Integral Equations Operator Theory* **82** (2015), no. 1, 61–94; Erratum: **85** (2016), no. 2, 301–302
- [78] F. Gesztesy, Y. Latushkin, K. A. Makarov, F. Sukochev, and Y. Tomilov, [The index formula and the spectral shift function for relatively trace class perturbations.](#) *Adv. Math.* **227** (2011), no. 1, 319–420
- [79] F. Gesztesy, Y. Latushkin, M. Mitrea, and M. Zinchenko, [Nonselfadjoint operators, infinite determinants, and some applications.](#) *Russ. J. Math. Phys.* **12** (2005), no. 4, 443–471. For a corrected and considerably updated version of Sections 4 and 5 of this paper, see 2020, arXiv:[math/0511371](https://arxiv.org/abs/math/0511371)
- [80] F. Gesztesy, Y. Latushkin, F. Sukochev, and Y. Tomilov, [Some operator bounds employing complex interpolation revisited.](#) In *Operator semigroups meet complex analysis, harmonic analysis and mathematical physics*, pp. 213–239, Oper. Theory Adv. Appl. 250, Birkhäuser/Springer, Cham, 2015
- [81] F. Gesztesy, M. Malamud, M. Mitrea, and S. Naboko, [Generalized polar decompositions for closed operators in Hilbert spaces and some applications.](#) *Integral Equations Operator Theory* **64** (2009), no. 1, 83–113
- [82] F. Gesztesy and R. Nichols, [On absence of threshold resonances for Schrödinger and Dirac operators.](#) *Discrete Contin. Dyn. Syst. Ser. S* **13** (2020), no. 12, 3427–3460
- [83] F. Gesztesy and R. Nichols, [Trace ideal properties of a class of integral operators.](#) In *Integrable systems and algebraic geometry. Vol. 1*, pp. 13–37, London Math. Soc. Lecture Note Ser. 458, Cambridge University Press, Cambridge, 2020
- [84] F. Gesztesy and B. Simon, [Topological invariance of the Witten index.](#) *J. Funct. Anal.* **79** (1988), no. 1, 91–102

- [85] I. C. Gohberg and M. G. Kreĭn, *Introduction to the theory of linear nonselfadjoint operators*. Transl. Math. Monogr. 18, American Mathematical Society, Providence, RI, 1969
- [86] I. C. Gohberg and M. G. Kreĭn, *Theory and applications of Volterra operators in Hilbert space*. Transl. Math. Monogr. 24, American Mathematical Society, Providence, RI, 1970
- [87] S. Golénia and T. Jecko, [A new look at Mourre's commutator theory](#). *Complex Anal. Oper. Theory* **1** (2007), no. 3, 399–422
- [88] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*. Academic Press, New York, 1980
- [89] J. C. Guillot and G. Schmidt, [Spectral and scattering theory for Dirac operators](#). *Arch. Rational Mech. Anal.* **55** (1974), 193–206
- [90] M. Hansmann, *On the discrete spectrum of linear operators in Hilbert spaces*. Ph.D. thesis, Technical University of Clausthal, Germany, 2005
- [91] M. Hansmann, [Perturbation determinants in Banach spaces—with an application to eigenvalue estimates for perturbed operators](#). *Math. Nachr.* **289** (2016), no. 13, 1606–1625
- [92] I. W. Herbst, [Spectral theory of the operator \$\(p^2 + m^2\)^{1/2} - Ze^2/r\$](#) . *Comm. Math. Phys.* **53** (1977), no. 3, 285–294
- [93] T. Ichinose, [Kato's inequality and essential selfadjointness for the Weyl quantized relativistic Hamiltonian](#). *Proc. Japan Acad. Ser. A Math. Sci.* **64** (1988), no. 10, 367–369
- [94] A. Iftimovici and M. Măntoiu, [Limiting absorption principle at critical values for the Dirac operator](#). *Lett. Math. Phys.* **49** (1999), no. 3, 235–243
- [95] H. T. Ito, [High-energy behavior of the scattering amplitude for a Dirac operator](#). *Publ. Res. Inst. Math. Sci.* **31** (1995), no. 6, 1107–1133
- [96] A. Jensen, [Spectral properties of Schrödinger operators and time-decay of the wave functions results in \$L^2\(\mathbf{R}^m\)\$, \$m \geq 5\$](#) . *Duke Math. J.* **47** (1980), no. 1, 57–80
- [97] A. Jensen, [Spectral properties of Schrödinger operators and time-decay of the wave functions. Results in \$L^2\(\mathbf{R}^4\)\$](#) . *J. Math. Anal. Appl.* **101** (1984), no. 2, 397–422
- [98] A. Jensen and T. Kato, [Spectral properties of Schrödinger operators and time-decay of the wave functions](#). *Duke Math. J.* **46** (1979), no. 3, 583–611
- [99] A. Jensen and G. Nenciu, [A unified approach to resolvent expansions at thresholds](#). *Rev. Math. Phys.* **13** (2001), no. 6, 717–754; Erratum: **16** (2004), no. 5, 675–677
- [100] H. Kalf, [The virial theorem in relativistic quantum mechanics](#). *J. Functional Analysis* **21** (1976), no. 4, 389–396
- [101] H. Kalf, [Nonexistence of eigenvalues of Dirac operators](#). *Proc. Roy. Soc. Edinburgh Sect. A* **89** (1981), no. 3-4, 309–317
- [102] H. Kalf, T. Okaji, and O. Yamada, [Absence of eigenvalues of Dirac operators with potentials diverging at infinity](#). *Math. Nachr.* **259** (2003), 19–41
- [103] H. Kalf, T. Okaji, and O. Yamada, [The Dirac operator with mass \$m_0 \geq 0\$: non-existence of zero modes and of threshold eigenvalues](#). *Doc. Math.* **20** (2015), 37–64; Erratum: **24** (2019), 1361–1363

- [104] H. Kalf, T. Okaji, and O. Yamada, A note on uniform resolvent estimates of Dirac operators. *Mem. Inst. Sci. Engrg. Ritsumeikan Univ.* (2015), no. 74, 1–9
- [105] H. Kalf and O. Yamada, Note on the paper: “Strong unique continuation property for the Dirac equation” by L. De Carli and T. Okaji. *Publ. Res. Inst. Math. Sci.* **35** (1999), no. 6, 847–852
- [106] H. Kalf and O. Yamada, Essential self-adjointness of n -dimensional Dirac operators with a variable mass term. *J. Math. Phys.* **42** (2001), no. 6, 2667–2676
- [107] T. Kato, Wave operators and similarity for some non-selfadjoint operators. *Math. Ann.* **162** (1965/66), 258–279
- [108] T. Kato, *Perturbation theory for linear operators. Corr. printing of the 2nd edn.* Grundlehren Math. Wiss. 132, Springer, Cham, 1980
- [109] H. Kitada, Scattering theory for the fractional power of negative Laplacian. *J. Abstr. Differ. Equ. Appl.* **1** (2010), no. 1, 1–26
- [110] H. Kitada, A remark on simple scattering theory. *Commun. Math. Anal.* **11** (2011), no. 2, 124–138
- [111] M. G. Krein, On perturbation determinants and the trace formula for unitary and selfadjoint operators. *Soviet Math. Dokl.* **3** (1962), 707–710
- [112] S. T. Kuroda, *An introduction to scattering theory*. Lect. Notes Ser. 51, Matematisk Institut, Aarhus Universitet, Aarhus, 1978
- [113] H. Leinfelder, A remark on a paper: “A compactness condition for linear operators of function spaces” by L. D. Pitt. *Bayreuth. Math. Schr.* (1982), no. 11, 57–66
- [114] E. H. Lieb and M. Loss, *Analysis*. 2nd edn., Grad. Stud. Math. 14, American Mathematical Society, Providence, RI, 2001
- [115] M. Loss and B. Thaller, Short-range scattering in long-range magnetic fields: the relativistic case. *J. Differential Equations* **73** (1988), no. 2, 225–236
- [116] M. Loss and H.-T. Yau, Stability of Coulomb systems with magnetic fields. III. Zero energy bound states of the Pauli operator. *Comm. Math. Phys.* **104** (1986), no. 2, 283–290
- [117] M. Macheda, On the Birman–Schwinger principle applied to $\sqrt{-\Delta + m^2} - m$. *J. Math. Phys.* **47** (2006), no. 3, Paper No. 033506
- [118] M. Măntoiu and M. Pascu, Global resolvent estimates for multiplication operators. *J. Operator Theory* **36** (1996), no. 2, 283–294
- [119] L. Mattner, Complex differentiation under the integral. *Nieuw Arch. Wiskd. (5)* **2** (2001), no. 1, 32–35
- [120] R. C. McOwen, The behavior of the Laplacian on weighted Sobolev spaces. *Comm. Pure Appl. Math.* **32** (1979), no. 6, 783–795
- [121] K. Mochizuki, On the perturbation of the continuous spectrum of the Dirac operator. *Proc. Japan Acad.* **40** (1964), 707–712
- [122] M. Murata, Asymptotic expansions in time for solutions of Schrödinger-type equations. *J. Funct. Anal.* **49** (1982), no. 1, 10–56

- [123] B. Najman, Scattering for the Dirac operator. *Glasnik Mat. Ser. III* **11(31)** (1976), no. 1, 63–80
- [124] R. G. Newton, *Scattering theory of waves and particles*. Dover Publications, New York, 2002
- [125] L. Nirenberg and H. F. Walker, The null spaces of elliptic partial differential operators in \mathbf{R}^n . *J. Math. Anal. Appl.* **42** (1973), 271–301
- [126] T. Okaji, Absence of eigenvalues of Dirac type operators. In *Partial differential equations and mathematical physics (Tokyo, 2001)*, pp. 157–176, Progr. Nonlinear Differential Equations Appl. 52, Birkhäuser, Boston, MA, 2003
- [127] T. Okaji, On the spectrum of dirac operators. *RIMS Kôkyûroku, Kyoto Univ.* **1607** (2008), 65–76
- [128] B. P. Palka, *An introduction to complex function theory. Corr. 2nd printing*. Undergrad. Texts Math., Springer, New York, 1995
- [129] M. Persson, Zero modes for the magnetic Pauli operator in even-dimensional Euclidean space. *Lett. Math. Phys.* **85** (2008), no. 2-3, 111–128
- [130] L. D. Pitt, A compactness condition for linear operators of function spaces. *J. Operator Theory* **1** (1979), no. 1, 49–54
- [131] C. Pladdy, Asymptotics of the resolvent of the Dirac operator with a scalar short-range potential. *Analysis (Munich)* **21** (2001), no. 1, 79–97
- [132] C. Pladdy, Resolvent estimates for the Dirac operator in weighted Sobolev spaces. *Asymptot. Anal.* **31** (2002), no. 3-4, 279–295
- [133] C. Pladdy, Y. Saitō, and T. Umeda, Resolvent estimates of the Dirac operator. *Analysis* **15** (1995), no. 2, 123–149
- [134] C. Pladdy, Y. Saitō, and T. Umeda, Radiation condition for Dirac operators. *J. Math. Kyoto Univ.* **37** (1997), no. 4, 567–584
- [135] D. Potapov and F. Sukochev, Unbounded Fredholm modules and double operator integrals. *J. Reine Angew. Math.* **626** (2009), 159–185
- [136] R. T. Prosser, Relativistic potential scattering. *J. Mathematical Phys.* **4** (1963), 1048–1054
- [137] A. Pushnitski, The spectral flow, the Fredholm index, and the spectral shift function. In *Spectral theory of differential operators*, pp. 141–155, Amer. Math. Soc. Transl. Ser. 2 225, American Mathematical Society, Providence, RI, 2008
- [138] A. G. Ramm, Perturbation of resonances. *J. Math. Anal. Appl.* **88** (1982), no. 1, 1–7
- [139] M. Reed and B. Simon, *Methods of modern mathematical physics. II. Fourier analysis, self-adjointness*. Academic Press, New York, 1975
- [140] M. Reed and B. Simon, *Methods of modern mathematical physics. IV. Analysis of operators*. Academic Press, New York, 1978
- [141] M. Reed and B. Simon, *Methods of modern mathematical physics. I. Functional analysis*. 2nd edn., Academic Press, New York, 1980

- [142] S. Richard, *Some improvements in the method of the weakly conjugate operator.* *Lett. Math. Phys.* **76** (2006), no. 1, 27–36
- [143] S. Richard and T. Umeda, *Low energy spectral and scattering theory for relativistic Schrödinger operators.* *Hokkaido Math. J.* **45** (2016), no. 2, 141–179
- [144] H.-W. Rohde, *Ein Kriterium für das Fehlen von Eigenwerten elliptischer Differentialoperatoren.* *Math. Z.* **112** (1969), 375–388
- [145] S. N. Roze, *On the spectrum of the Dirac operator.* *Theoret. Math. Phys.* **2** (1970), 275–279
- [146] G. Rozenblum and N. Shirokov, *Infiniteness of zero modes for the Pauli operator with singular magnetic field.* *J. Funct. Anal.* **233** (2006), no. 1, 135–172
- [147] M. Ruzhansky and M. Sugimoto, *Structural resolvent estimates and derivative nonlinear Schrödinger equations.* *Comm. Math. Phys.* **314** (2012), no. 2, 281–304
- [148] O. Safronov, *Spectral shift function in the large coupling constant limit.* *J. Funct. Anal.* **182** (2001), no. 1, 151–169
- [149] O. Safronov, *Absolutely continuous spectrum of a Dirac operator in the case of a positive mass.* *Ann. Henri Poincaré* **18** (2017), no. 4, 1385–1434
- [150] Y. Saitō and T. Umeda, *The asymptotic limits of zero modes of massless Dirac operators.* *Lett. Math. Phys.* **83** (2008), no. 1, 97–106
- [151] Y. Saitō and T. Umeda, *The zero modes and zero resonances of massless Dirac operators.* *Hokkaido Math. J.* **37** (2008), no. 2, 363–388
- [152] Y. Saitō and T. Umeda, *Eigenfunctions at the threshold energies of magnetic Dirac operators.* *Rev. Math. Phys.* **23** (2011), no. 2, 155–178
- [153] Y. Saitō and T. Umeda, *A sequence of zero modes of Weyl–Dirac operators and an associated sequence of solvable polynomials.* In *Spectral theory, function spaces and inequalities*, pp. 197–209, Oper. Theory Adv. Appl. 219, Birkhäuser/Springer Basel AG, Basel, 2012
- [154] K. M. Schmidt, *Spectral properties of rotationally symmetric massless Dirac operators.* *Lett. Math. Phys.* **92** (2010), no. 3, 231–241
- [155] K. M. Schmidt and T. Umeda, *Spectral properties of massless Dirac operators with real-valued potentials.* In *Spectral and scattering theory and related topics*, pp. 25–30, RIMS Kôkyûroku Bessatsu, B45, Res. Inst. Math. Sci. (RIMS), Kyoto, 2014
- [156] K. M. Schmidt and T. Umeda, *Schnol’s theorem and spectral properties of massless Dirac operators with scalar potentials.* *Lett. Math. Phys.* **105** (2015), no. 11, 1479–1497
- [157] B. Simon, *Quantum mechanics for Hamiltonians defined as quadratic forms.* Princeton Ser. Phys., Princeton University Press, Princeton, NJ, 1971
- [158] B. Simon, *Phase space analysis of simple scattering systems: extensions of some work of Enss.* *Duke Math. J.* **46** (1979), no. 1, 119–168
- [159] B. Simon, *Trace ideals and their applications.* 2nd edn., Math. Surveys Monogr. 120, American Mathematical Society, Providence, RI, 2005
- [160] E. M. Stein, *Singular integrals and differentiability properties of functions.* No. 30 in Princeton Math. Ser., Princeton University Press, Princeton, NJ, 1970

- [161] B. Thaller, Potential scattering of Dirac particles. *J. Phys. A* **14** (1981), no. 11, 3067–3083
- [162] B. Thaller, Normal forms of an abstract Dirac operator and applications to scattering theory. *J. Math. Phys.* **29** (1988), no. 1, 249–257
- [163] B. Thaller, Dirac particles in magnetic fields. In *Recent developments in quantum mechanics (Poiana Brașov, 1989)*, pp. 351–366, Math. Phys. Stud. 12, Kluwer Acad. Publ., Dordrecht, 1991
- [164] B. Thaller, Scattering theory of a supersymmetric Dirac operator. In *Differential equations with applications in biology, physics, and engineering (Leibnitz, 1989)*, pp. 313–326, Lecture Notes in Pure and Appl. Math. 133, Dekker, New York, 1991
- [165] B. Thaller, *The Dirac equation*. Texts Monogr. Phys., Springer, Berlin, 1992
- [166] B. Thaller and V. Enss, Asymptotic observables and Coulomb scattering for the Dirac equation. *Ann. Inst. H. Poincaré Phys. Théor.* **45** (1986), no. 2, 147–171
- [167] M. Thompson, Eigenfunction expansions and the associated scattering theory for potential perturbations of the Dirac equation. *Quart. J. Math. Oxford Ser. (2)* **23** (1972), 17–55
- [168] M. Thompson, The absence of embedded eigenvalues in the continuous spectrum for perturbed Dirac operators. *Boll. Un. Mat. Ital. A* (5) **13** (1976), no. 3, 576–585
- [169] E. Toprak, A weighted estimate for two dimensional Schrödinger, matrix Schrödinger, and wave equations with resonance of the first kind at zero energy. *J. Spectr. Theory* **7** (2017), no. 4, 1235–1284
- [170] C. Tretter, *Spectral theory of block operator matrices and applications*. Imperial College Press, London, 2008
- [171] H. Triebel, *Interpolation theory, function spaces, differential operators*. 2nd edn., Johann Ambrosius Barth, Heidelberg, 1995
- [172] T. Umeda, Absolutely continuous spectra of relativistic Schrödinger operators with magnetic vector potentials. *Proc. Japan Acad. Ser. A Math. Sci.* **70** (1994), no. 9, 290–291
- [173] T. Umeda, Radiation conditions and resolvent estimates for relativistic Schrödinger operators. *Ann. Inst. H. Poincaré Phys. Théor.* **63** (1995), no. 3, 277–296
- [174] T. Umeda, The action of $\sqrt{-\Delta}$ on weighted Sobolev spaces. *Lett. Math. Phys.* **54** (2000), no. 4, 301–313
- [175] T. Umeda, Generalized eigenfunctions of relativistic Schrödinger operators. I. *Electron. J. Differential Equations* **2006** (2006), Paper No. 127
- [176] T. Umeda and M. Nagase, Spectra of relativistic Schrödinger operators with magnetic vector potentials. *Osaka J. Math.* **30** (1993), no. 4, 839–853
- [177] T. Umeda and D. Wei, Generalized eigenfunctions of relativistic Schrödinger operators in two dimensions. *Electron. J. Differential Equations* **2008** (2008), Paper No. 143
- [178] K. Veselić and J. Weidmann, Existenz der Wellenoperatoren für eine allgemeine Klasse von Operatoren. *Math. Z.* **134** (1973), 255–274
- [179] V. Vogelsang, Absence of embedded eigenvalues of the Dirac equation for long range potentials. *Analysis* **7** (1987), no. 3-4, 259–274

- [180] V. Vogelsang, *Absolutely continuous spectrum of Dirac operators for long-range potentials*. *J. Funct. Anal.* **76** (1988), no. 1, 67–86
- [181] G. N. Watson, *A treatise on the theory of Bessel functions*. 2nd edn., Cambridge University Press, Cambridge, 1966
- [182] D. Wei, *Completeness of the generalized eigenfunctions for relativistic Schrödinger operators. I*. *Osaka J. Math.* **44** (2007), no. 4, 851–881
- [183] J. Weidmann, *Linear operators in Hilbert spaces*. Grad. Texts in Math. 68, Springer, New York, 1980
- [184] D. R. Yafaev, *Mathematical scattering theory. General theory*. Transl. Math. Monogr. 105, American Mathematical Society, Providence, RI, 1992
- [185] D. R. Yafaev, *A trace formula for the Dirac operator*. *Bull. London Math. Soc.* **37** (2005), no. 6, 908–918
- [186] D. R. Yafaev, *Mathematical scattering theory. Analytic theory*. Math. Surveys Monogr. 158, American Mathematical Society, Providence, RI, 2010
- [187] O. Yamada, *On the principle of limiting absorption for the Dirac operator*. *Publ. Res. Inst. Math. Sci.* **8** (1972/73), 557–577
- [188] O. Yamada, *Eigenfunction expansions and scattering theory for Dirac operators*. *Publ. Res. Inst. Math. Sci.* **11** (1975/76), no. 3, 651–689
- [189] O. Yamada, *A remark on the limiting absorption method for Dirac operators*. *Proc. Japan Acad. Ser. A Math. Sci.* **69** (1993), no. 7, 243–246
- [190] Y. Zhong and G. L. Gao, *Some new results about the massless Dirac operator*. *J. Math. Phys.* **54** (2013), no. 4, Paper No. 043510