Preface

Die Schriftsteller
auch wenn sie Wissenschaftler sind
sind Übertreibungsspezialisten

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These notes are based on graduate ('Nachdiplom') lectures given in spring term 2022 at the Department of Mathematics, ETH Zürich, Switzerland. I am very grateful to the Forschungsinstitut für Mathematik (FIM) at ETH Zürich for hosting me, as well as to Afonso Bandeira and Sara van der Geer for their hospitality during my visit.

Chapters 1 and 2 develop a framework of Bayesian Gaussian process methods in non-linear random design regression models and give sufficient conditions to obtain global convergence guarantees for posterior measures in PDE-type inverse problems. Chapters 3 to 5 develop the local theory about fluctuations and shape of posterior measures in high dimensions, and touch on the related issue of convergence properties of gradient-based MCMC algorithms. The reader will require background in real analysis, measure-theoretic probability and stochastic convergence theory – one may consult [46] or similar texts. We will frequently use mathematical techniques from high-dimensional statistics and probability as developed in [60]. Relevant material from elliptic partial differential equations, functional analysis and stochastic calculus will be reviewed in Appendices A and B.

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