Abstract

In the ordinary theory of Sobolev spaces on domains of $\mathbb{R}^n$, the p-energy is defined as the integral of $|\nabla f|^p$. In this paper, we try to construct a p-energy on compact metric spaces as a scaling limit of discrete p-energies on a series of graphs approximating the original space. In conclusion, we propose a notion called conductive homogeneity under which one can construct a reasonable p-energy if p is greater than the Ahlfors regular conformal dimension of the space. In particular, if $p = 2$, then we construct a local regular Dirichlet form and show that the heat kernel associated with the Dirichlet form satisfies upper and lower sub-Gaussian type heat kernel estimates. As examples of conductively homogeneous spaces, we present new classes of square-based self-similar sets and rationally ramified Sierpiński crosses, where no diffusions were constructed before.

In memory of the late Professors Robert S. Strichartz and Ka-Sing Lau,
who were my dearest friends and daring explorers of the frontiers

Keywords. Sobolev spaces, metric spaces, conductive homogeneity, p-energy, self-similar sets

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