Abstract

A formalism of arithmetic partial differential equations (PDEs) is being developed in which one considers several arithmetic differentiations at one fixed prime. In this theory solutions can be defined in algebraically closed p-adic fields. As an application we show that for at least two arithmetic directions every elliptic curve possesses a non-zero arithmetic PDE Manin map of order 1; such maps do not exist in the arithmetic ODE case. Similarly, we construct and study "genuinely PDE" differential modular forms. As further applications we derive a Theorem of the kernel and a Reciprocity theorem for arithmetic PDE Manin maps and also a finiteness Diophantine result for modular parameterizations. We also prove structure results for the spaces of "PDE differential modular forms defined on the ordinary locus." We also produce a system of differential equations satisfied by our PDE modular forms based on Serre and Euler operators.

Keywords. Arithmetic differential equations, modular forms, quasi-canonical lifts, overconvergence

Mathematics Subject Classification (2020). Primary 11F32; Secondary 11F85, 11G07, 11G18

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