

# Contents

|   |     |
|---|-----|
| <b>Preface</b> . . . . .  | vii |
| <b>Introduction</b> . . . . .   | 1   |
| <b>1 Euclidean apartments</b> . . . . .                                   | 9   |
| 1.1 Groups generated by reflections and apartments . . . . .              | 9   |
| 1.2 Linear reflection groups . . . . .                                    | 15  |
| 1.3 General reflection groups . . . . .                                   | 26  |
| 1.4 Discrete reflection groups . . . . .                                  | 35  |
| 1.5 Facets and general apartments . . . . .                               | 52  |
| 1.6 Appendix: Root systems . . . . .                                      | 62  |
| <b>2 Euclidean buildings</b> . . . . .                                    | 77  |
| 2.1 Definitions and general properties . . . . .                          | 77  |
| 2.2 Discrete Euclidean buildings . . . . .                                | 95  |
| 2.3 Metric properties of Euclidean buildings . . . . .                    | 107 |
| 2.4 Types, products and completion . . . . .                              | 125 |
| <b>3 Properties at infinity</b> . . . . .                                 | 139 |
| 3.1 Sector-friendly systems of apartments . . . . .                       | 139 |
| 3.2 Spherical building at infinity . . . . .                              | 155 |
| 3.3 Chimney-friendly buildings . . . . .                                  | 176 |
| 3.4 Polyhedral compactification . . . . .                                 | 193 |
| <b>4 Groups of automorphisms</b> . . . . .                                | 209 |
| 4.1 Fixed point theorems . . . . .  | 209 |
| 4.2 Strongly transitive actions . . . . .                                 | 215 |
| 4.3 Moufang buildings . . . . .   | 228 |
| 4.4 Affine Moufang buildings . . . . .                                    | 242 |
| <b>5 Buildings associated to groups</b> . . . . .                         | 255 |
| 5.1 Parabolic groups associated to RGD systems . . . . .                  | 255 |
| 5.2 Parahoric groups associated to valued RGD systems . . . . .           | 270 |
| 5.3 Decompositions . . . . .  | 286 |
| 5.4 Construction of buildings with a strongly transitive action . . . . . | 295 |
| 5.5 Morphisms . . . . .   | 309 |

|   |     |
|---|-----|
| <b>6 Examples . . . . .</b>   | 327 |
| 6.1 Vectorial buildings associated to $GL_n$ . . . . .              | 327 |
| 6.2 Affine buildings associated to $GL_n$ . . . . .                 | 343 |
| 6.3 Vectorial buildings associated to isometry groups . . . . .     | 375 |
| 6.4 Affine buildings associated to isometry groups . . . . .        | 413 |
| 6.5 Buildings as ultralimits . . . . .                              | 437 |
| <b>7 Applications . . . . .</b>                                     | 453 |
| 7.1 Retractions of apartments, galleries or line segments . . . . . | 453 |
| 7.2 LS paths and Hecke paths . . . . .                              | 475 |
| 7.3 Hecke algebras . . . . .  | 493 |
| 7.4 Iwahori–Hecke algebras . . . . .                                | 500 |
| 7.5 Spherical Hecke algebras . . . . .                              | 528 |
| 7.6 Discreteness . . . . .  | 551 |
| <b>A Notations . . . . .</b>  | 569 |
| A.1 Use of letters . . . . .  | 569 |
| A.2 Mathematical conventions . . . . .                              | 570 |
| <b>References . . . . .</b>   | 573 |
| <b>Notation index . . . . .</b>                                     | 585 |
| <b>Subject index . . . . .</b>                                      | 589 |