

Contents

1 General overview	1
1.1 Historical context	1
1.2 Mathematical reformulation and objectives	3
1.3 Cluster expansion formalism	7
1.4 Main results	14
1.5 Roadmap to the main results	17
Notation	19
2 Einstein's formula: First-order expansion	21
2.1 Main result	21
2.2 Variational approach	24
2.3 Preliminary lemmas	29
2.4 Proof of Lemma 2.3	36
2.5 Proof of Lemma 2.4	38
2.6 Explicit form of Einstein's formula	41
3 Cluster expansion of the effective viscosity	43
3.1 Finite-volume approximations	43
3.2 Main results	44
3.3 Preliminary lemmas	47
3.4 Cluster formulas	51
3.5 Uniform $\ell^1 - \ell^2$ energy estimates	60
3.6 Uniform cluster estimates	75
3.7 Convergence of finite-volume approximations	75
3.8 Non-uniform cluster estimates	80
4 Renormalization of cluster formulas	85
4.1 Main results	85
4.2 Implicit renormalization of cluster formulas	89
4.3 Preliminary to explicit renormalization	90
4.4 Explicit renormalization of cluster formulas	97
4.5 Optimality of cluster estimates	142
5 Conclusion	147
5.1 Cluster expansion of the effective viscosity	147
5.2 Summability of the cluster expansion	150

A Stokeslet estimates with rigid inclusions	155
A.1 Main results	155
A.2 Decay of Stokeslets with rigid inclusions	158
A.3 Mean-value property with rigid inclusions	163
A.4 Periodization errors	166
B Finite-volume approximation of the effective viscosity	173
References	183