THE HUG OF THE SCUTOID

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ABSTRACT

This paper is a personal account of my work in the popularization of mathematics. How I started doing math popularization, why I think it is important to do this kind of tasks, and, finally, how this work can lead to some fruitful results in pure research that, initially, seems not to be related with that work of popularization.

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1. INTRODUCTION

Day after day, the popularization and dissemination of the results obtained by the researchers is not only considered more important, but it is compulsory in order to fulfill the conditions of many research grants. Many reasons have been given to justify why science popularization and dissemination is crucial. In fact, nowadays, as it is said in [4], "Dissemination and communication of research should be considered as an integral part of any research project. Both help in increasing the visibility of research outputs, public engagement in science and innovation, and confidence of society in research. Effective dissemination and communication are vital to ensure that the conducted research has a social, political, or economical impact. They draw [the] attention of governments and stakeholders to research results and conclusions, enhancing their visibility, comprehension, and implementation." But this paper tries to be just a personal account of my experience in the field of math popularization, and one thing is why universities, governments, and any other institution must encourage dissemination and popularization of science in general, or mathematics in particular, and a very different thing is why any particular individual, myself in this occasion, is doing this kind of tasks. Of course, in order to understand a particular case, we have to keep in mind a more general scope, so, we shall briefly try to answer the typical questions of why, what, where, and how, both from a general and from my very personal point of view. In fact, those topics, to a greater or lesser extent, have been considered previously in the six works on this subject (just six) in other ICM's editions. In this way, Ian Stewart's work [9] considers where, and he analyzes the many possible types of media which can be used for popularization. He focuses on magazines, newspapers, books, radio, and television, but the internet is missing, so, eight years later, Etienne Ghys [1] focused precisely on the role of the internet. A general perspective was discussed in a panel directed by Günter Ziegler [11] and seeking the same goal is the purpose of the first work on this subject presented at ICM [8]. Finally, the other two articles are mainly concerned with one of those big questions, and so [3] tries to give clues about "what" and [6] is focused on "who."

In this work, I am going to try to answer some of those big questions, but mainly treating the "why" one. So, firstly I will tell why I started to work on math popularization and why I think it is important, adding a couple or reasons to those more commonly given by general institutions. For instance, a committee of the British government on strategies for education (see http://nationalstrategies.standards.dcsf.gov.uk/node/16073) addresses the importance of mathematics in society:

> *"Mathematical thinking is important for all members of a modern society as a habit of mind for its use in the workplace, business and finance, and for personal decision-making. Mathematics is fundamental to national prosperity in providing tools for understanding science, engineering, technology and economics. It is essential in public decision-making and for participation in the knowledge economy."*

But those arguments apply only to institutions, not to individuals. Hardly anyone works in any subject, even less in one as math popularization with a difficult rentabilization from the point of view of the academic career, with those high targets in mind. But, something interesting I have learned from my personal perspective is that we can add some additional arguments to those usually given.

The structure of this paper is as follows. Firstly, in the next section, I will tell how and why I started doing math popularization and will show my personal point of view of the other general questions we have mentioned before. I think it is important to point out that one of my main concerns is how to bring mathematics closer to women and girls, and the other way around: how to bring women and girls closer to mathematics, and why I think this is important. Section 3 reports an unexpected benefit of my work in math popularization and provides an additional reason to those usually given by institutions, since we have found how that task is important in order to obtain new results in basic science, even more important, in a very multidisciplinary work. We will finish with some conclusions and plans for future works.

2. WHO, WHAT, WHERE, WHEN, WHY, AND HOW (MY PERSONAL VIEW)

Several reasons have been given to encourage science popularization in general, and mathematics in particular. The rapid development and extensive application of science and technology since 1870 not only promoted a profound transformation in economy and society, but also deeply impacted the mode of production, people's lifestyle, and the basic relationship between science and the public. But it is difficult for the general public to understand the role of mathematics in that development, so almost all institutions try to impel the spreading of a certain knowledge of that role in people's lifestyle. But, I have to confess that I did not get up one morning thinking "mm, I must spread my math knowledge because that is important for society." Not at all, In my case, after obtaining my PhD degree (with a thesis in Computational Geometry), I was lucky enough that a well-known publisher contacted me and so I, and my advisor, embarked into transforming the thesis into a book for that publisher. In the meantime, I continued with my research, trying to publish some papers and sending works to conferences. This is to say, I did not care about math popularization at all; at least, as an actor. So, what was the starting point of my career in this field? Well, I have to confess that after the book two kids came, and with them a lot of questions (after a while, of course). Many of those questions referred to mathematics, simply because their dad and mum are mathematicians. I tried to answer those questions, and their dad encouraged me to write those answers in a personal blog I kept at that time. Nobody read that blog before that entry, but, suddenly those explanations became quite popular and I got thousands of clicks. In one way or the other, that post was read by an illustrator (Raquel García Ulldemolins) and, simultaneously, a very popular blog in that time asked me to collaborate with them. Someone suggested that Raquel and I could make a tandem, so, I wrote the text and she painted some illustrations. In this way, "Mati y sus mateaventuras" (Mati and her mathadventures) was born.

The structure of all the posts in that blog is similar: two brothers (Sal and Ven) are arguing about any subject and then, they meet their friend Mati and she shows then that the

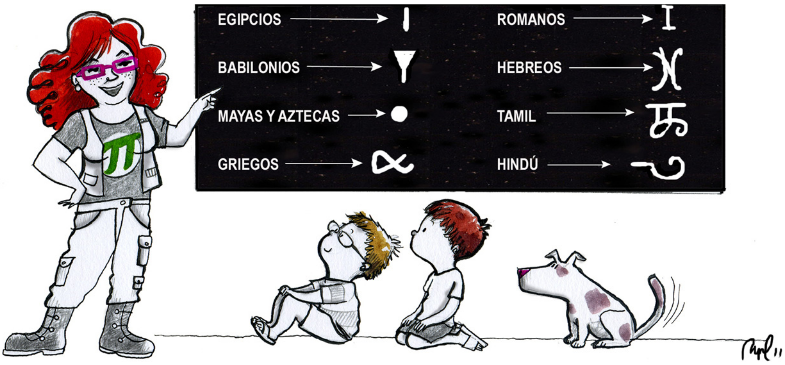

FIGURE 1
One of the illustrations in the first post of “Mati y sus mateaventuras”.

subject is full of mathematical facts; and with them their dog (Gauss is its name) is always given a humorous counterpoint (see Figures 1 and 2).

I have to say that this blog had quite an impact and obtained several awards. Initially, we had in mind young people as the target of our stories, but we soon realized that a lot of parents and, especially, teachers started to use the ideas we were showing there as an inspiration or as a pedagogical material for their families or in their classrooms. So, right after the blog, many visits to schools came and, after that, talks in many different places (universities, secondary schools, science museums, even bars, and cafeterias), radio, books, television, etc.

Thus, after all this activity, I can say that my main professional activity in the last ten years has been math popularization. I think in all this process I have learned some things about this discipline. Firstly, if we assume that math popularization is important, it is crucial the support of the institution but, at the same time, we have to try to carry our message to traditional media because a good portion of our possible and desirable target does not access yet to the new (or not so new) places where an important part of the popularization activity is done (around internet, mainly). On the other hand, the institutions must create the adequate climate to foster the work of those who decide to dedicate a part of their time to try to convince the society that mathematics is important (as all the other branches of knowledge, of course). I am not an expert on that side of the equation, but three things can be done: firstly, universities must have their own units in order to organize activities open to the public, with a clear plan ahead and with measurable milestones; additionally, those units must help the members of the university doing actions in this field by giving technical support. Secondly (and in some way, related to point one), it is important to dedicate funds to these tasks. On many occasions, the people doing math popularization do it at their own expense. Lastly, the elements needed to value this work must be established. Not a long time ago, we did math popularization without telling our departments we were doing such a task after work, fearing that other members of those departments could think we were wasting our time.

But now, I think we can focus on the “what” side of popularization. Again, at least from my point of view. Of course, it is impossible to talk about “what” without taking into account the audience (“Who” is your public). It is not the same to give a talk in a bar to adult people as in a school to six-year-old students or a twenty minute slot in a radio show. Nevertheless, there exists a rule of the thumb for any activity: we have to keep in mind that, with very few exceptions, our audience is not very fond of mathematics and, in many cases, they think that the discipline is useful, but they cannot give examples of its usefulness other than elementary arithmetic calculations. So, our central thread must be an application of mathematics, if possible a very unexpected one. Or, at least, to show a puzzle challenging enough to engage and inspire your audience. Then, with the excuse of solving the application we have presented (or the puzzle), we can (we must) the beautiness of the involved mathematics. Of course, the application does not need to be a crucial one for humanity. For instance, in one of the posts, I wrote about one of the few mathematical articles that appeared in Nature [5]. In that work, the author tries to find, under different criteria, the best way to lace the shoes. Or in another one, I reproduce the simple computations needed to solve a very important problem: how to leave the toilet seat after using it (males mainly). And an obvious answer is “clean,” of course, following this, more academic work: https://www.scq.ubc.ca/a-game-theoretic-approach-to-the-toilet-seat-problem/. Well, probably those two examples are, indeed, crucial for humanity.

In any case, it is clear, I think, that “What,” “Who,” and “How” are closely interrelated (and even “Where”), and so every time we try to communicate something about mathematics, we have to keep in mind those three factors and to adapt our message under those conditions.

A side note about “How.” In my case, I always use the same style, with the Raquel García Ulldemolins’s illustrations (with the exception of the radio, of course) and a certain naive touch.

2.1. Why mathematics?

Briefly, I would like to point out three reasons (among hundreds) to try to answer the question “Why is it important to make math popular?”. At least, those are the factors I have in mind when preparing any action on this subject, especially giving a talk or arranging any other activity in schools.

Firstly, to fight against some myths (“mathematics is only for few people (and they are nerds)”, “common people only need to know elementary arithmetic operations,” “I am not fit for mathematics”, etc.). The main problem here is that in an elementary school (and even in a secondary school) in many countries, mathematics is taught just as a tedious repetition of some computations without putting them in context. So, it is important here to show how mathematics is present in many facts of everyday life, and why having a certain knowledge and understanding of mathematics can help when we make some decisions.

Secondly, for the students, the analytic mind that we can train with the proper problems in mathematics can help in almost all the other subjects. And finally, I have noticed the positive value of a talk in those gifted students (and in many cases, they do not know about

FIGURE 2
After some time, the characters in “Mati y sus mateaventuras” were changing a little bit, but preserving the same flavor.

their capacities). There are some studies in this area and, for example, in [10], the authors show, as they pointed out in the abstract, that within the same society, adolescent students who specifically lack mathematical education exhibited reduced brain inhibition levels in a key brain area involved in reasoning and cognitive learning. Importantly, these brain inhibition levels predicted mathematical attainment 19 months later, suggesting they play a role in neuroplasticity. And their study provides a biological understanding of the impact of the lack of mathematical education on the developing brain and the mutual interplay between biology and education. We will see something about the mutual interplay between biology and mathematics later.

2.2. Where are the women?

The second half of the last century has seen an increase of women with mathematical degrees, until reaching the 50% level in many countries and even more than that, but we have seen that this process has been reversed in this century. As Figure 3 shows, the turning point was around the beginning of this century and it coincides (causality or not, we as mathematicians know pretty well that correlation does not imply causality) with the moment when data, information, algorithms, and many other subjects related with mathematics entered our common life with a high demand for mathematicians in many companies. In other words, this happened when teaching became not the first option after obtaining the degree.

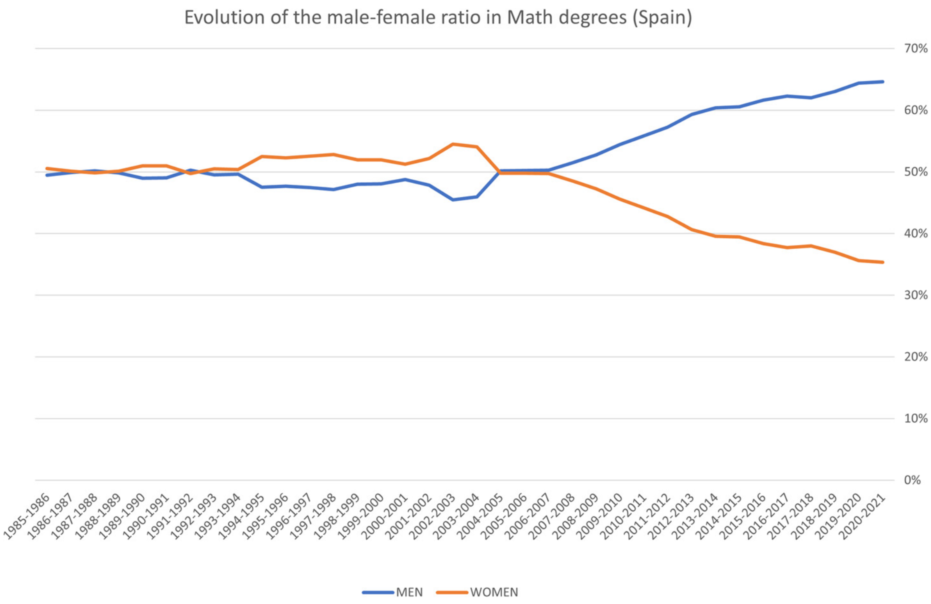

FIGURE 3

One can observe the effect of "scisors" in the evolution of the male–female ratio in math degrees in Spain. The line representing women grows until the year 2000, and then it decreases.

I think it is worthy to copy what is said[1] about this problem and its possible solutions:

> "Male scientists outnumber females two to one, [...]. According to the National Girls Collaborative Project, 'Women make up half of the total U. S. college-educated workforce, but only 29% of the science and engineering workforce.'
>
> Girls are interested in math and science when they're young, but they're often diverted before high school and eventually declare their college major in another field. An American Association of University Women study of 1,226 female science professionals found that girls actually demonstrate interest in science at a young age, but are discouraged due to antagonistic, critical behavior in many math and science departments. Nearly 40 percent of respondents indicated experiencing such behavior.
>
> Maybe the problem isn't gender-based but in the way children's skills are fostered. Math Professor Mary Beth Ruskai argues that both boys and girls need more interactions with scientists to become interested in science. Schools should also identify and encourage students' talents, regardless of academic field. Educational reform efforts often yield increased retention rates for both males and females, simultaneously combating two problems.

1 https://www.learningliftoff.com/encourage-girls-math-and-science-courses-and-careers/.

Here are a few ways teachers and parents can keep that spark of interest in science going for young girls and encourage more women scientist in the future:

1. Create Projects Based on Interest

Instead of letting girls' math and science interests lie dormant or go ignored, we should present them with science projects based on their interest. Sometimes, all it takes is one successful project to give a girl the encouragement she needs to find her passion in math or science. Even some toys for young children can aid in kindling an interest in science.

2. Introduce Female Math and Science Role Models

The U. S. Department of Commerce reports that women hold only 24 percent of STEM occupations, and those with a STEM (Science, Technology, Engineering, and Math) degree typically work in education or healthcare. While the numbers seem bleak, it presents an opportunity for change. Females working in or holding degrees in math or science should serve as role models for girls seeking a career in their field. Introducing a positive role model of the same gender to young girls can keep them interested and have a lifelong impact on their career paths.

3. Emphasize the Positives

Parents and educators should encourage girls to defy the stereotypes that math and science are only for boys. Like any subject, if girls are struggling in math and science, teachers should help them work through their struggles. This can mean playing an active role in helping them better understand these subjects. Just because a female student finds the subjects difficult is not a reason to move away from the field entirely. Working through challenges is part of the learning experience. Confidence plays a large role in a girl's success in science and math, and it's important to help her maintain a high level of confidence.

4. Explore Career Options Early

Often, kids in elementary and high school are unaware of the myriad career options that will be available to them as adults. Many of these careers will require a broader background in subjects they may not have considered or cultivated an interest in. But if they are able to explore career interests early, they can better prepare for them by taking classes they might have avoided otherwise. A student may want to be a doctor or a veterinarian, for example, but not be aware of the important role that science will take for such careers. And some job fields, such as computer coding and programming, encourage students to begin training in high school and even elementary school to be truly competitive. If career education courses are not offered in your child's school, consider an alternative school choice such as an online career academy. Destinations Career Academies and Programs combine traditional high school academics with career education.

In short, science and math are not gender-specific fields, yet girls seem to tune out natural tendencies toward these subjects. We can change their attitudes toward math and science by offering them encouragement, role models, and opportunities to learn, tapping into their innate scientific and math skills."

I think that we, from an external point of view, can relate to those four points, and it is a good guide to follow.

3. AND, SUDDENLY, FLIES

In this section, I am going to try to show an additional benefit of math popularization, based on a personal experience. This benefit is not for the general public, for society, but mainly for other researchers. I will try to show how the popularization of mathematics can help in some interdisciplinary fields, building bridges between other sciences and mathematics, and fostering new and important results.

3.1. Voronoi diagram and 2-dimensional epithelial tissues

As "The New Yorker" says in an article,[2] "One of the many mysteries of living cells is how they manage to blossom into coherent many-celled units. A person or a platypus begins as a single cell, which divides into more cells, which also divide and subdivide. Some of these, the epithelial cells, are destined to become tissues and organs. The cells collect into layers, which bend and fold into greater-than sums: ovaries, kidneys, a heart. In part, it's a packing challenge, a geometry problem; as the layers twist and curve, the individual cells change shape in accordance with the whole, and they do so as efficiently as possible."

So, the research group of Luisma Escudero was focused on the problem of describing the shape of the epithelial cells. In fact, they had made some progress in the 2-dimensional case (when the cells are very flat). And their result was based on the Voronoi diagram. According to Wikipedia, "A Voronoi diagram is a partition of a plane into regions close to each of a given set of objects. In the simplest case, these objects are just finitely many points in the plane (called seeds, sites, or generators). For each seed, there is a corresponding region, called a Voronoi cell, consisting of all points of the plane closer to that seed than to any other." In this way, Luisma Escudero and his group modeled the shape of those 2-dimensional cells using the following method.

Firstly, they generate a set of random points in a plane region, then they compute the Voronoi diagram of that set of points, this can be considered the iteration 0. For next iterations, they compute the centroid of each Voronoi region obtained in the previous iteration and compute, again, the Voronoi diagram of that set of centroids. In this way, they conclude that a good model for this kind of tissues is obtained after five iterations. This means that by comparing some parameters (number cells with a given number of neighbors, quantity of some structures, etc.) of a sample with their model tissue, they can check if the sample corresponds to a healthy individual or if it presents some problem [7] (see Figure 4).

2 https://www.newyorker.com/science/lab-notes/we-are-all-scutoids-a-brand-new-shape-explained.

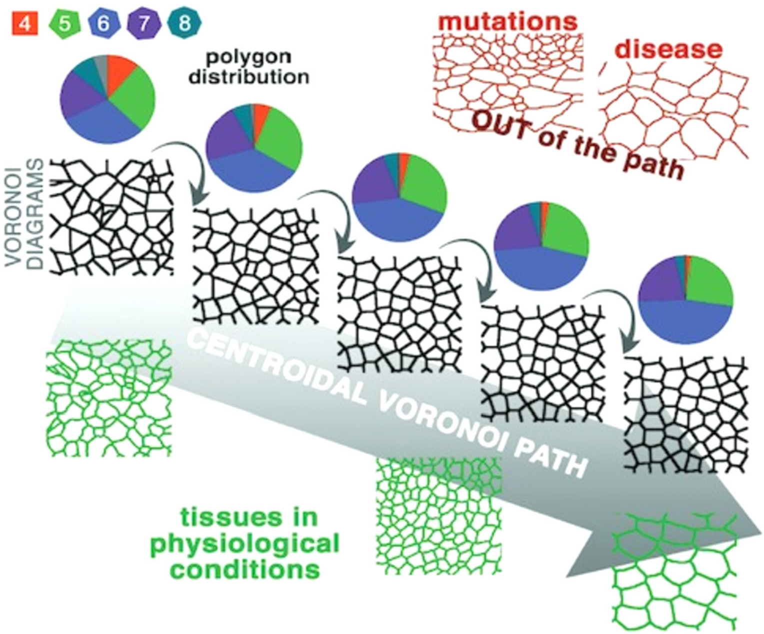

FIGURE 4
A synopsis of the results obtained in [7].

But, as animals develop, tissue bending contributes to shaping the organs into complex three-dimensional structures. However, the architecture and packing of curved epithelia remains largely unknown, and it was known that the results for the 2-dimensional model are not valid anymore. Thus, the transition from planar epithelial sheets to cylindrical, ellipsoidal, or spherical forms involves a fundamental reorganization of the cells.

It must be known that an epithelial tissue must be thought as a sheet and all the cells in that tissue appear on both sides of the tissue (called the apical and basal surfaces). Thus, in all the books of Cellular Biology, those cells are represented by prisms or truncated pyramids. But a close examination under microscope shows that the neighboring cells are not the same on the apical and basal surfaces. So, another model for those cells was needed.

3.2. Scutoids and 3-dimensional epithelial tissues

Of course, the first idea is to consider some variation of Voronoi diagrams, but the problem is that the model must predict what really happens in epithelial tissues, and Escudero's group was stuck. Then, he read a couple of articles I had written for one of the most important platform for science popularization in Spain (Naukas) about Voronoi Diagrams (https://naukas.com/2011/12/23/cada-uno-en-su-region-y-voronoi-en-la-de-todos/ and https://naukas.com/2012/01/28/esta-voronoi-que-se-ponga/) and he decided to contact me. That was the beginning of a beautiful (and fruitful) collaboration.

After some failures, we finally modeled the scutoids, following the following steps:

Regarding the space, we start with a given surface S, then for each point $X(u.v) \in S$, we consider the normal vector to S at $X(u, v)$, $N(u, v)$. Thus, for each $\lambda \in [0, 1]$, it is possible to define a new surface S_λ parallel to S in such a way that any point of S_λ is $X_\lambda(u, v) = X(u, v) + \lambda N(u, v)$ (a point in one of the surfaces has an equivalent point in each one of the other parallel surfaces). The metric in each surface previously defined is just the distance of the shortest geodesic on that surface joining two points. As it is well known, in the case of the cylinder, the geodesics are the helices in the cylinder.

We define every seed starting in a point on the apical surface. That point defines a segment between the basal and apical surfaces by means of its normal (given the point $X(u, v)$, the segment is $X(u, v) + \lambda N(u, v)$, $\lambda \in [0, 1]$). The intersection of these line segments with a given surface determines a seed. Thus, in order to generate all the seeds, in a first step we had chosen n points on the apical surface, then the n segments that were generated by them, and, finally, the intersection of those segments with every surface S_λ defined the seeds for that surface.

The next step is to compute the Voronoi diagrams of the seeds obtained in each one of the parallel surfaces. We linked the Voronoi regions corresponding to the seeds on the same segment, obtaining a three-dimensional figure, which we called a *scutoid* (see Figure 5).

3.3. Almost famous

After the publication of our results in Nature Communications, we made an important work in the dissemination of its results and scutoids appeared everywhere. Of course, in the most important media such as "New York Times," "The New Yorker," "The Times," as well as in the news of most of the TV companies (BBC, CBS, Fox, etc.), and in some shows as "The Late Show with Stephen Colbert." This led artists, designers, architects, and engineers to produce some works in their fields based on the shape and properties of the scutoids. In order to achieve this success, we sent press releases, called press conferences, and, in general, we talked about scutoids everywhere and everytime.

4. THE SHOW MUST GO ON. CONCLUSIONS AND FUTURE WORK

Although the beginning of my work in math popularization started in a unplanned way, it has reached such a volume that, I think, I must concentrate on the things I believe are more productive: firstly, encourage girls to study mathematics or any other science, or obtain a technical degree, and secondly, try to spread the usefulness of mathematics to a very broad public by using some media, such as the radio, that are not so popular in math popularization (it is quite a challenge to describe the shape of a scutoid if we have no images).

But, just as at the beginning, I really do not know what I am going to do in five years from now. Let us see.

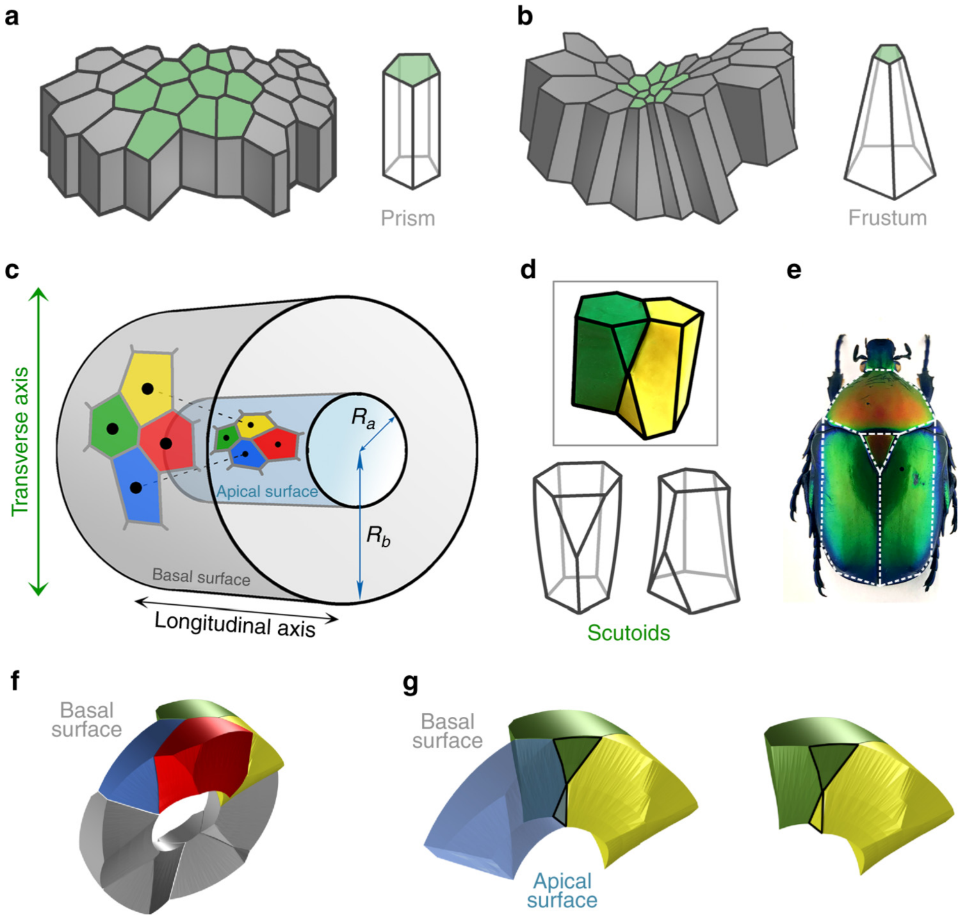

FIGURE 5
A graphical synopsis of the results obtained in [2].

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