SOME USES AND ASSOCIATIONS OF MATHEMATICS, AS SEEN FROM A DISTANT HISTORICAL PERSPECTIVE

ANNETTE IMHAUSEN

ABSTRACT

The article presents the evolution of mathematics and its various uses in ancient Egypt.

MATHEMATICS SUBJECT CLASSIFICATION 2020

Primary 01A16; Secondary 01A11, 01A80

KEYWORDS

History of mathematics, ancient Egypt

INTERNATIONAL CONGRESS OF MATHEMATICIANS 2022 JULY 6–14

Proc. Int. Cong. Math. 2022, Vol. 7, pp. 5772–5782
DOI 10.4171/ICM2022/170

Published by EMS Press

1. INTRODUCTION

Today mathematics constitutes an integral part of our lives on a variety of levels. For those present at this conference (and many others), the discipline of mathematics is their chosen profession which is subdivided in a multitude of individual areas. In addition, scientists other than those of the mathematical sciences also use its concepts and methods in their professional lives whenever they work with data that can be represented numerically. Thus, the education in these sciences often includes mathematical training that is specifically tailored to their needs. Furthermore, mathematics also plays an important role in everybody's daily life using numerical practices and mathematical measuring in many contexts, for example, in structuring the day by means of measuring time or by quantifying the many things like items of food or other objects. Although, as many mathematicians have encountered from nonmathematicians when asked what they do for a living, some seem to take pride in "always having been bad at mathematics." But even for these people, it would be hard to imagine them getting by without it. The unenthusiastic statement thus may less reflect on the subject itself than on the methods used in teaching it. Several attempts to present the appeal of mathematics to a general audience have succeeded, for example, the volume *The Mathematical Experience* (1981) by Philip Davis and Reuben Hersh [4] which won the National Book Award in 1983. These days, mathematics influences various areas of our lives as has been demonstrated with numerous examples by the Cambridge mathematician Tom Körner in his book *The Pleasures of Counting* (1996) [8]. While the examples chosen by Tom Körner mostly originate from a more modern context the history of mathematics has been traced back to begin as early as in the 3rd millennium BCE with the invention of writing in Egypt and Mesopotamia.

My own research has always focused on ancient Egyptian mathematics, which can provoke an ambivalent reaction from historians of mathematics. On the one hand, Egypt (along with Mesopotamia) provides one of the earliest types of evidence for mathematical concepts and techniques in history. On the other hand, ancient Egyptian mathematics has been judged by some of its more recent readers as lagging behind its potential, and the occasional judgment went as far as blaming their clumsy mathematical techniques for a lack of development in other areas that would have depended on them. While the history of mathematics was initially often performed as a judgmental assessment of mathematical sources read from a modern point of view, (most) historians and (most) mathematicians alike have since realized that it is much more rewarding to attempt learning about mathematical concepts of a historic period as they were used and practiced in their time. It may be trivial to solve an ancient mathematical problem with a modern mathematical toolbox, and thus equally boring. However, to learn how this problem was thought of in its time and tackled with the then available tools is as complex as it is rewarding. This has consequences for the methodology deemed appropriate in the history of mathematics. It also has consequences for the evaluation of ancient mathematical cultures. While this has been the focus of some of my earlier work [6], this contribution has a different aim, trying to bridge gaps between periods and cultures. Because then as today the practice of mathematics has had multiple aspects as described in the initial sentences and it is through an analysis of these aspects that parallels as well as differences between ancient and modern mathematics can be seen. My contribution will look at some of these aspects from the mathematics of ancient Egypt.

2. INVENTION OF NUMBER SYSTEM AND ITS EARLY USES

The earliest evidence we have for ancient Egyptian numbers comes from a graveyard in Abydos. The stratification of society of ancient Egypt can be traced by the sizes and contents of tombs in graveyards. Cemetery U at Abydos, situated to the north of the Early Dynastic royal tombs, contains approximately 600 tombs and covers the entire Predynastic period; see Hartung [5, P. 317]. The tomb Uj of that cemetery has become rather well known in Egyptology for the about 100 incised tags that were found within it. The incisions were either pictograms resembling later hieroglyphic signs or groups of strokes or a coiled rope resembling later Egyptian number notation. All of the tags had a hole, which points to them having been attached to some sort of goods about which they would provide information of some kind. They are considered the first evidence of writing from ancient Egypt; see Baines [1]. Despite the difficulties of interpreting these earliest sources, the tags may point to a parallel to Mesopotamia in the invention of writing and the number system, where this was prompted by administrative needs; see Robson [15]. In case of the tags found in the tomb Uj, it is possible to interpret them as some sort of administrative practice denoting, for instance, owner or provenance and quantities, a practice that was presumably first used in daily life and then as well in the context of burials. At this point it is noteworthy that as in Mesopotamia, the first evidence of writing from ancient Egypt includes as well the first evidence of numbers.

Evidence from the Old Kingdom indicates that the use of what we think of as mathematics was located in the context of administration. While there are no mathematical texts from this period in ancient Egypt, two archives of that time show that numbers and metrological units were used to audit resources of larger structures like temples; see the work of Posener-Kriéger and Demichelis [12,13]. Later evidence points to an ongoing tradition of this administrative context for the practice and development of mathematics. The Lahun Papyri, the largest papyri find from the Middle Kingdom, include a large number of accounts; see Collier and Quirke [3]. Evidence from all periods of ancient Egypt indicates that numbers, metrology, and mathematical practices were used to monitor and audit resources of various kinds (goods as well as manpower). The Egyptian systems of numeration and metrologies constitute the basis of this administrative oversight.

However, even then there were further uses of numbers. An early object that displays the Egyptian number system is the ritual mace-head of king Narmer, the last king of dynasty 0 (c. 3000 BCE). The decoration of the mace-head includes the picture of a tribute of 400,000 oxen, 1,422,000 smaller cattle, and 120,000 prisoners that were presented to king Narmer. The mace-head illustrates that the number system with its symbols to indicate powers of ten up to one million was in use as early as in dynasty 0. The large numbers indicated on the mace-head likely do not indicate quantities of an actual tribute, but were probably meant to represent the power of this king through their size. These first early examples of numbers both originate from a royal context. The restricted use of writing and numbers in the service of the Egyptian king and his institutions is another characteristic that has to be kept in mind when analyzing the mathematics of ancient Egypt.

In the control of the king's resources, a professional group emerged whose most common name denotes its crucial skill: scribes. Scribes were active in the administration. The complex system of responsibilities is reflected in a variety of individual titles that are attested since the Old Kingdom; see Jones [7]. The basic requirement for the profession of scribe was the ability to read and write. It can be assumed that this was passed down from father to son. Likewise, it can be assumed that "writing" included not only the handling of script but also the handling of numbers. The following sections of this paper will outline the use of mathematics by scribes in the Old, Middle, and New Kingdoms as well as mathematical applications in Egyptian culture beyond the realm of administration.

3. THE OLD KINGDOM: EARLY EVIDENCE OF MATHEMATICAL CONCEPTS AND PRACTICES

The first period that is recognized as a period of remarkable cultural achievements in ancient Egypt is the Old Kingdom (c. 2686–2160 BCE). During the Old Kingdom, Egyptian kings were buried in pyramids. Several kings of the 5th Dynasty chose an area northwest of the modern village of Abusir for their pyramids. In the late 19th century, papyri were purchased by several museums that came from the administration of the cult for one of these kings. Taken together, these papyri constitute the largest papyrus find from the Old Kingdom. The Abusir papyri contain, among other things, accounts and papyri from the cult operations that took place there, such as service lists of priests, ration lists, lists of sacrifices for temples, and others. They are the oldest preserved papyri that are extant. They also contain information about the further development of Egyptian mathematics in the form of the creation of metrological systems. The ration lists contain quantities of grain, meat, and beer, each given in the corresponding units. Also attested in these papyri is the use of tables, which are recognizable as such not only by tabular arrangement of entries, but also by formatting with rows and columns marked by line and column headings. From these texts we obtain information about the activity of the scribes who, at temples or at the royal court, prepared these accounts and lists and through their work performed the administration of the empire, its resources and especially its goods produced at the lower levels.

Further information about these persons, who were responsible for the control of important parts of the Egyptian productions, for example, of bread and beer, by means of mathematical techniques, can be obtained from their tombs. There we find representations of the production processes, such as the bakery and brewery, in which the scribes overseeing them are a central part. For example, in the tomb of Nianchchnum and Chnumhotep, the production of bread and beer is depicted from the allocation of the grain to the delivery of the products; see Moussa and Altenmüller [10, PLATES 23, 26, 34–35]. When the grain is allotted, it is shown how one person measures the flour, another counts the amount of flour that is taken, a scribe keeps a record of it, and one person receives it. This is followed by the depiction of the production of bread and beer from the flour. Finally, the delivered products are measured and noted by a scribe. In the depiction of the delivery, the bakers are shown bringing the products, as well as an auditing process, the result of which is noted down by a scribe.

This detailed audit of resources is also evident in the later mathematical texts of the Middle Kingdom. In these texts, a large number of problems deal with the management and control of resources; in particular, the number of bread-and-beer-problems, which at 19 accounts for slightly less than 1/5 of all mathematical problems, is striking. Despite the lack of evidence in form of mathematical texts of the Old Kingdom, the papyri and representation of daily life practices provide ample evidence for the further advancing of mathematics in ancient Egypt. Metrological systems for a variety of goods were created and used and thus a mathematical record of resources established.

Apart from the depictions of scribes in the context of executing numerical processes, the tombs of high-ranking officials also contain the so-called autobiographies of the tomb-owner. Ancient Egyptian autobiographies differ fundamentally from modern autobiographies. An ancient Egyptian autobiography refers to a text written in the 1st person singular describing the career and proper actions of the tomb's owner. It served to vindicate the tomb owner and represented a guarantee that he would pass the judgment of the dead by proving that he had lived his life according to the moral principles of ancient Egypt. Consequently, no negative events appear in the autobiography, the path of life is described as a continuous ascent to ever new responsibilities. In comparison to modern autobiography, personal areas of life, such as family and the expression of emotions, are missing.

Egyptian autobiographies are attested throughout pharaonic history and allow us to trace changes in the perception and self-assessment of the scribes' profession. By describing the professional activities of scribes, they also provide evidence for the role of mathematics in the life of a scribe. For the Old Kingdom, the autobiography of Weni from the 6th Dynasty is one of the most remarkable examples—not least because Weni served successively under three pharaohs, Teti, Pepi I, and Merenre. Weni is thought to have been active for a period of 70 years, his career must have begun in his youth. The text of his autobiography was written in carefully executed hieroglyphics on a stone slab that once formed the wall of a one-room burial chapel in Abydos (for a translation cf. Lichtheim [9, PP. 18–23]). The autobiography of Weni lists services performed by Weni for the king and rewards that Weni received in return. These rewards were either in the form of material goods or in the form of a promotion to a higher position. Particular emphasis is placed on activities in which Weni acted entirely on his own—demonstrating a special form of trust in him by the king. In this autobiography, explicit reference is made at least once to mathematical activities. Weni says of himself that he counted everything that could be counted in Upper Egypt twice for the king and that he counted every activity twice that had to be done for the Upper Egyptian residence as well. Through their accounting, the scribes monitored all kinds of resources for their king which placed themselves in a position of power. During the Old Kingdom, the autobiographies document that the level of success of a scribe was measured by his proximity to the king whom he served.

Summing up, the evidence from the Old Kingdom indicates the proliferation of using mathematics in the organization of resources and the evolution of a professional group that used mathematical practices. Some members of that group held exalted positions within Egyptian society as is indicated by the size and endowment of their tombs.

During the First Intermediate Period (2160–2055 BCE), the central rule of Egypt by a single king broke down. This breakdown seems to have been affected by several causes, which taken together could not be overcome by the former royal authority. The Egyptian literature of the following Middle Kingdom successfully attempted to reestablish royal authority by presenting how the new kings had overcome this dark age and its difficulties. The description of the First Intermediate Period given from a later royal point of view has led Egyptologists initially to assess this period as a dark one, associated with social and political instabilities, an assessment that probably holds for some years of the First Intermediate Period. Its beginning, with famines caused by climatic changes and the failure of the former king to maintain control over all of Egypt, must have been a drastic change for the Egyptian population. However, the autobiographies of some local nomarchs indicate that these problems were then mastered. During the First Intermediate Period, while the central administrative framework was lacking, the individual nomarchs presumably continued their administrative roles towards the population of their towns or regions. The individual success of a nomarch, as it is expressed in the autobiographies, was measured through his ability to ensure social and economic stability within his own region and through his conduct towards the weaker members of society. In the work of an official, mathematical knowledge must have played an important role in order to assess (for example) the available grain rations. Using this knowledge, however, was no longer perceived as a service to the king. Instead, the nomarchs saw themselves as installed through the power of the gods.

4. THE DISCIPLINE OF MATHEMATICS IN THE MIDDLE KINGDOM (AND AFTER)

As is well known, several papyri about mathematics are extant from the Middle Kingdom (c. 2055–1650 BCE) and the Second Intermediate Period (c. 1650–1550 BCE). The most famous of them are the Rhind Mathematical Papyrus [11] and the Moscow Mathematical Papyrus [16]. The Rhind papyrus, named after Alexander Henry Rhind, the Scottish lawyer who purchased the text, is kept today in London in the British Museum. The Moscow Mathematical Papyrus is named after the city of its current location in the Pushkin State Museum of Fine Arts.

Their initial appearance at this point in time may, of course, be due to the haphazard circumstances of preservation. However, they may also reflect the conscious attempt of the Middle Kingdom rulers to reestablish control over administrative structures that they had lost during the First Intermediate Period. Egyptian mathematical texts may contain mathematical problems and their solutions and mathematical tables used for fraction reckoning and conversion of measures. The Rhind Mathematical Papyrus, being in two pieces, has the inventory numbers BM 10057 and BM 10058. BM 10057 measures 295.5 cm by 32 cm, BM 10058 199.5 cm by 32 cm. The gap between both pieces is assumed to be approximately 18 cm. The Rhind Mathematical Papyrus provides a corpus of some 70 problems and several tables. Most of the problems are grouped together according to their content.

The Moscow Mathematical Papyrus is the second largest extant source text. While its total length is approximately 544 cm, its height is only 8 cm. It consists of one big piece and nine little fragments. The Moscow Papyrus contains 25 problems, of which the first three are too damaged to determine more than a probable type of problem. In addition, unlike the problems of the Rhind Papyrus, those of the Moscow papyrus were not arranged in groups of problems according to their content but seem to be written down in no apparent order. The Moscow papyrus also holds two duplicate problems in numbers 8 (identical with problem no. 5) and 13 (identical with problem no. 9). However, among the problems of the Moscow papyrus are two of the most interesting for historians of mathematics, problem no. 10 about the area of a curved surface and problem no. 14 about the volume of a truncated pyramid.

Egyptian mathematical problem texts are written in a distinct style: A problem text begins by stating a mathematical problem (title). After the type of problem is announced, some specific data in the form of numerical values are given, thus specifying the problem to one concrete instance or object. This is followed by instructions (the procedure) for its solution. Title, and specifications of the problem and the following instructions are expressed in prose, using no mathematical symbolism. The title (and other parts of the text) may be accentuated by the use of red ink. Each instruction usually consists of one arithmetic operation (addition, subtraction, multiplication, division, halving, squaring, extraction of the square root, calculation of the inverse of a number) and the result of it is given. The instructions always use the specific numerical values assigned to the problem. Abstract formulas, or equations with variables did not exist. This style, which is also used in Mesopotamian problem texts, has been characterized by Jim Ritter as rhetoric, numeric, and algorithmic [14, P. 44].

Historiographic assessment of Egyptian mathematics has followed two paths so far, first the description of the mathematics found in these texts by means of modern mathematical terminology. In this line of inquiry, Egypt was praised to provide very early evidence for "algebraic equations" and other early mathematical knowledge, like that of calculating the area of certain geometric shapes. In comparison with contemporary evidence from Mesopotamia, however, it fell short and, compared with later evidence from ancient Greece, it was lacking the feature of general mathematical theorems and their proofs. The second, more sophisticated line of inquiry of research on ancient Egyptian (and Mesopotamian) mathematics attempts to understand mathematical structures within the source texts, e.g., by assessing and comparing the procedures used in solving mathematical problems. Again, Mesopotamia has fared somewhat better than Egypt, which may, however, be the result of the very different quantities of sources available. This line of inquiry is not yet exhausted at this point.

The contents of the problems enforce the impression that the context of ancient Egyptian mathematics remained within the area of administration. However, while most of the problems can be understood as mathematical solutions to actual administrative tasks, several problems point to an awareness of mathematical knowledge as such, for example, problem 79 of the Rhind Papyrus that asks to compute the total of a number of items comprised of a house and cats, mice and cereal found within. Likewise, the phrasing of problem 67 of the Rhind Papyrus, ostensibly computing the produce of a herdsman, points to the existence of a different type of mathematical setting, comparable with so-called recreational mathematics.

Due to the fragmentary state of preservation, only two titles of mathematical papyri are known. Of these two, one fits the assessment of mathematics as a tool in administration. Fragment UC32162 of the Lahun Mathematical Papyri contains fragments of two problems, a calculation of areas and a calculation of the produce of a fowler. Before the text of these problems a title reading "[Method] of calculating the matters of accounting" is extant. The other title is found at the beginning of the Rhind Mathematical Papyrus, and reads "Method of calculating for inquiring into nature, and for knowing all that exists, [every] mystery, [...] every secret" which seems to point to an appreciation of mathematical knowledge that exceeds its simple utility in administration.

Thus, while the disciplinary context of mathematics in ancient Egypt remains within administration, the recognition that its application may be wider than the accounting of resources is indicated by the content and title of the Rhind Papyrus. Sources from later periods provide examples of these further applications.

5. FURTHER USES OF MATHEMATICAL CONCEPTS AND PRACTICES IN ANCIENT EGYPT

After the Second Intermediate Period there is a lack of sources as far as mathematical texts are concerned. However, mathematics still features quite prominently in the lives of the scribes as literary texts indicate. Instead of school books of individual subjects, a variety of texts, which were presumably circulated among New Kingdom scribes, is extant. They include compositions describing the superiority of the scribal profession over any other profession, eulogies to scribal teachers, and model letters. This corpus of texts is referred to as the *Late Egyptian Miscellanies*. At least some of these texts include references to mathematical practices. The theme of scribal superiority above all other professions is the topic of the following excerpt, section 4,2–5,7 of Papyrus Lansing, which was titled "All callings are bad except that of the scribe" by its first translator, Ricardo Caminos [2, pp. 384–385]. Reference to scribal work is made twice within this section, first when describing how the profit of the merchants is taken away by the tax-people (scribes!), and again at the end of the passage when the profession of the scribe is compared to the aforementioned professions: "But the scribe, it is he that reckons the produce of all those." Both of these references are with respect to the mathematical abilities of the scribe, who has to calculate the taxes of the merchants before carrying them off and who also calculates the output of the other professions, presumably to determine their taxes. These references provide evidence for the ongoing use of mathematics in administration as well as the gains that those proficient in it were to expect. Likewise, mathematics features in the text of Papyrus Anastasi I, a fictional letter from the context of a learned debate between two scribes. The debate includes a set of mathematical problems: the calculation of bricks needed to construct a ramp; the number of workers needed to transport an obelisk; the number of workers needed to move sand when a colossal statue has to be erected in a given time; and the calculation of rations for a military excursion. Although these problems are phrased like their earlier counterparts of the mathematical texts, the numerical information given in Papyrus Anastasi I does not suffice to actually solve these problems. Their intention may have been to remind the numerate reader of his mathematical education. To us, the text is a source that provides us with an idea of the variety of numerate tasks that a scribe had to master. In addition, it also informs us about areas of their profession that scribes thought of as meaningful and valuable. Thus, although there is practically no evidence for mathematical texts, administrative documents and evidence from literary texts leave no doubt that mathematics continued to play an important role in the scribal profession during the New Kingdom.

In addition, by this time mathematics had also acquired another function in ancient Egyptian society. Not only did it provide the means to perform administrative tasks, but it did so in a way that was considered to fulfill the requirements of acting according to the Egyptian moral code. The normative framework of this moral code is expressed by the Egyptian term *Maat*. This term comprises the idea, that there is a certain perfect order of the cosmos and everything in it. Therefore, the term *Maat* is closely linked or may express ideas of truth, order and justice. From the idea of a perfect order of the cosmos then follow certain codes of conduct to which every Egyptian was supposed to adhere in his or her daily life. For some literate members of Egyptian society, the respective rules were explicitly stated in a genre of ancient Egyptian literature called teachings or instructions. Extant Teachings (with settings from the Old Kingdom on) provide examples for scribes (*Loyalist Teaching*, *Teaching of Khety*, *Instruction of Any* and *Instruction of Amenemope*), viziers (*Instruction addressed to Kagemni* and *Teaching of the Vizier Ptahhotep*), princes (*Instruction of Prince Hardjedef*) or even kings (*Teaching for King Merikare*, *Teaching of King Amenemhat*). From the four examples of teachings addressed to scribes, *The Teaching of Amenemope*, includes several references to mathematics beginning with the introduction of its fictional author Amenemope, which identifies him as the person who controls the measuring of agricultural affairs including the registration of land and audit of the vessels used to measure grain. The authority of the scribe, formally provided by his being in the service of the king, de facto originates from his numerical and metrological expertise, which enable him to execute the tasks mentioned in his introduction. Further references to numerical and metrological duties occur throughout the following 30 chapters of instructions. It is explicitly mentioned that a scribe must not "falsify the temple rations" (Chapter 5), "move the markers on the borders of fields, or shift the position of the measuring-cord," "be greedy for a cubit of land, or encroach on the boundaries of a widow" (Chapter 6), "move the scales or alter the weights, or diminish the fractions of the measure," "desire a measure of the fields, or neglect those of the treasury," "make for himself deficient weights" (Chapter 16), "disguise the measure, so as to falsify its fractions," and "force it (the measure) to overflow, or let its belly be empty" (Chapter 17). The teaching illustrates, on the one hand, the role that was by now assigned to mathematics, i.e., to provide justice, and, on the other hand, it indicates the awareness that consisted in a dishonest scribe who would falsify mathematical results. In depictions of metrological practices and administrative texts that document them, this awareness is also reflected, since it seems to have been the rule that there was rarely a single scribe entrusted with measuring and recording the respective results, but usually a group of scribes who would then check each other's work. In the context of this ensuring of justice, it is noteworthy that the issue of setting numerical values, e.g., the amounts of produce that were expected of a worker, is never discussed nor addressed explicitly. The king was simply expected to execute his power according to the rules of *Maat*.

Mathematical practices also were used in another context that provides evidence for the concurrence of mathematics and justice in ancient Egypt, namely the judgement of the dead. In order to prove worthy for the afterlife, the deceased had to pass judgement of his way of life. In order to be successful, the deceased first had to recite sins that he did not commit during his lifetime. Then his heart was weighed on a balance against a feather, a symbol of the goddess Maat. If the balance showed equilibrium between the heart and the feather, the judgement was passed successfully and the dead would be presented to Osiris. It was the mathematical operation of weighing that ensured a just judgement of the deceased.

6. CONCLUSION

From its first beginnings in Egypt and Mesopotamia, mathematics as a discipline has made immense progress, the history of which is traced in the history of mathematics. If this is done in a historically correct way, it can provide fascinating glimpses into a variety of mathematical cultures. At the same time, mathematics has at all times remained a key element in our daily lives. The two aspects have always been intertwined.

In using mathematics for daily life purposes, the aspect of justice and reliability are key characteristics that can be demonstrated as early as in ancient Egypt, and presumably ever since. However, as history and especially the crisis of recent years indicate, using mathematics does not in itself guarantee success. Ancient Egyptian sources indicate an awareness of the potential weaknesses of measuring on an abstract level (as seen in the teachings) as well as in practice (as seen by the depiction of several scribes that are meant to perform the measuring and thus are meant to check each other). The success of mathematical practice therefore depended on two aspects, the quality of a mathematical technique that was developed to solve a given problem and the quality of its execution by its practitioners.

REFERENCES

[1] J. Baines, The earliest Egyptian writing: development, context, purpose. In *The first writing. Script invention as history and process*, edited by S. D. Houston, pp. 150–189, Cambridge University, Cambridge, 2004.

[2] R. A. Caminos, *Late-Egyptian miscellanies (Brown Egyptological Studies I)*. Oxford University Press, London, 1954.

[3] M. Collier and S. Quirke, *The UCL Lahun Papyri: accounts*. BAR. Int. Ser. 1471, Archaeopress, London, 2006.

[4] P. Davis and R. Hersh, *The mathematical experience*. Birkhäuser, Boston, 1981.

[5] U. Hartung, Cemetery U at Umm el-Qaab and the funeral landscape of the Abydos region in the 4th millennium BC. In *Desert and the Nile. Prehistory of the Nile Basin and the Sahara Papers in honour of Fred Wendorf*, edited by J. Kabaciński, M. Chłodnicki, M. Kobusiewicz, and M. Winiarska-Kabacińska, pp. 313–338, Stud. Afr. Archaeol. 15, Poznań Archaeological Museum, Poznań, 2018.

[6] A. Imhausen, *Mathematics in Ancient Egypt. A contextual history*. Princeton University Press, Princeton, 2016.

[7] D. Jones, *An index of Ancient Egyptian titles, epithets and phrases of the Old Kingdom*. Archaeopress, Oxford, 2000.

[8] T. W. Körner, *The pleasures of counting*. Cambridge University Press, Cambridge, 1996.

[9] M. Lichtheim, *Ancient Egyptian literature, Volume I: The Old and Middle Kingdoms*. University of California Press. Berkeley and Los Angeles, 1975.

[10] A. Moussa and H. Altenmüller, *Das Grab des Nianchchnum und Chnumhotep*. Philipp von Zabern, Darmstadt, 1977.

[11] T. E. Peet, In *The Rhind Mathematical Papyrus, British Museum 10057 and 10058*, The University Press of Liverpool and Hodder & Stoughton, London, 1923.

[12] P. Posener-Kriéger, *Les archives du temple funéraire de Néferirkaré-Kakaï: Les papyrus d'Abousir. Traduction et commentaire*. Bibl. Étude 65, Institut français d'archéologie orientale du Caire, Cairo, 1976.

[13] P. Posener-Kriéger and S. Demichelis, *I papiri di Gebelein: scavi G. Farina 1935*. 2004. *I papiri di Gebelein: scavi G. Farina 1935*. Turin: Ministero per i Beni e le Attività Culturali, Soprintendenza al Museo delle Antichità Egizie, Turin, 2004.

[14] J. Ritter, Chacun sa verité: les mathématiques en Égypte et en Mésopotamie. In *Éléments d'histoire des sciences*, edited by M. Serres, pp. 39–61, Bordas, Paris, 1989.

[15] E. Robson, Literacy, numeracy, and the state in early Mesopotamia: social and cultural practices. In *Literacy and the state in the Ancient Mediterranean*, edited by K. Lomas, R. D. Whitehouse, and J. B. Wilkins, pp. 37–50, Accordia Research Institute, London, 2007.

[16] W. W. Struve, *Mathematischer Papyrus des Staatlichen Museums der Schönen Künste in Moskau*. Quellen Stud. Gesch. Math., Abt. A: Quellen 1, Springer, Berlin, 1930.

ANNETTE IMHAUSEN

Arbeitsgruppe Wissenschaftsgeschichte, Historisches Seminar, Goethe-Universität Frankfurt, Norbert-Wollheim-Platz 1, 60323 Frankfurt am Main, Germany, annette.imhausen@normativeorders.net