Abstract

This is the first in a series of papers on projective positive energy representations of gauge groups. Let $\Xi \to M$ be a principal fiber bundle, and let $\Gamma_c(M, \mathrm{Ad}(\Xi))$ be the group of compactly supported (local) gauge transformations. If P is a group of "space–time symmetries" acting on $\Xi \to M$, then a projective unitary representation of $\Gamma_c(M, \mathrm{Ad}(\Xi)) \rtimes P$ is of *positive energy* if every "timelike generator" $p_0 \in \mathfrak{p}$ gives rise to a Hamiltonian $H(p_0)$ whose spectrum is bounded from below. Our main result shows that in the absence of fixed points for the cone of timelike generators, the projective positive energy representations of the connected component $\Gamma_c(M, \mathrm{Ad}(\Xi))_0$ come from 1-dimensional P-orbits. For compact M this yields a complete classification of the projective positive energy representations in terms of lowest weight representations of affine Kac–Moody algebras. For noncompact M, it yields a classification under further restrictions on the space of ground states.

In the second part of this series we consider larger groups of gauge transformations, which contain also global transformations. The present results are used to localize the positive energy representations at (conformal) infinity.