Contents

1 Introduction 1
- 1.1 Outline of Part I and II of this series 1
- 1.2 Structure of the present memoir 5
- 1.3 Connection to the existing literature 10

2 Projective representations of Lie groups 13
- 2.1 Locally convex Lie groups 13
- 2.2 Smooth representations 14
- 2.3 Central extensions 16
- 2.4 Integration of projective representations 18
- 2.5 Double extensions 20

3 Positive energy representations 21
- 3.1 Positive energy representations 21
- 3.2 Equivariant positive energy representations 22
- 3.3 Minimal representations 23
- 3.4 Cauchy–Schwarz estimates (general case) 26

4 Covariant extensions of gauge algebras 31
- 4.1 Gauge groups and gauge algebras 31
- 4.2 Reduction to simple structure algebras 34
- 4.3 Central extensions of gauge algebras 35

5 Cocycles for positive energy representations 39
- 5.1 Local gauge algebras 39
- 5.2 Infinitesimally free $\mathbb{R}$-actions 43
- 5.3 Cauchy–Schwarz estimates revisited 45

6 Continuity properties 47
- 6.1 Reduction to compact simple structure algebras 47
- 6.2 Extending the estimates from $D\mathfrak{g}$ to $\mathfrak{g}$ 49
- 6.3 Extending representations to Sobolev spaces 59
- 6.4 Sobolev–Lie algebras 64
- 6.5 The continuous extension theorem 70

7 The localization theorem 77
7.1 Statement and discussion of the theorem 77
7.2 Localization at the Lie algebra level 81
7.3 Twisted loop groups 84
7.4 Localization at the group level 88
7.5 Localization for equivariant representations 89

8 The classification for M compact 93
8.1 Positive energy representation of twisted loop groups 93
8.2 The classification theorem for compact base manifolds 99
8.3 Extensions to non-connected groups 102

9 The classification for M noncompact 105
9.1 Infinitely many lines 105
9.2 Infinitely many circles 113
9.3 A simple example with fixed points 121

A Twisted loop algebras and groups 125

B Twisted conjugacy classes in compact groups 127

C Restricting representations to normal subgroups 129

D Vacuum representations 131
D.1 Weak products and $\mathbb{R}$-actions 131
D.2 Vacuum representations 133

E Ergodic property of 1-parameter subgroups of $\widetilde{\mathrm{SL}}(2,\mathbb{R})$ 137

References 139

Index 147