

## References

- [1] N. Abatangelo and L. Dupaigne, [Nonhomogeneous boundary conditions for the spectral fractional Laplacian](#). *Ann. Inst. H. Poincaré C Anal. Non Linéaire* **34** (2017), no. 2, 439–467
- [2] N. Abatangelo and E. Valdinoci, [Getting acquainted with the fractional Laplacian](#). In *Contemporary research in elliptic PDEs and related topics*, pp. 1–105, Springer INdAM Ser. 33, Springer, Cham, 2019
- [3] R. P. D. Atkinson, C. J. Rhodes, D. W. Macdonald, and R. M. Anderson, [Scale-free dynamics in the movement patterns of jackals](#). *Oikos* **98** (2002), no. 1, 134–140
- [4] F. Bartumeus, J. Catalan, U. L. Fulco, M. L. Lyra, and G. M. Viswanathan, [Optimizing the encounter rate in biological interactions: Lévy versus Brownian strategies](#). *Phys. Rev. Lett.* **88** (2002), no. 9, article no. 097901. [Erratum](#). *Phys. Rev. Lett.* **89** (2002), no. 10, article no. 109902
- [5] K. Bogdan, T. Byczkowski, T. Kulczycki, M. Ryznar, R. Song, and Z. Vondraček, [Potential analysis of stable processes and its extensions](#). Lecture Notes in Math. 1980, Springer, Berlin, 2009. Edited by Piotr Graczyk and Andrzej Stos
- [6] C. Bucur and E. Valdinoci, [Nonlocal diffusion and applications](#). Lect. Notes Unione Mat. Ital. 20, Springer, Cham; Unione Matematica Italiana, Bologna, 2016
- [7] S. V. Buldyrev, E. P. Raposo, F. Bartumeus, S. Havlin, F. R. Rusch, M. G. E. da Luz, and G. M. Viswanathan, [Comment on “Inverse square Lévy walks are not optimal search strategies for  \$d \geq 2\$ ”](#). *Phys. Rev. Lett.* **126** (2021), no. 4, article no. 048901
- [8] L. Caffarelli, S. Dipierro, and E. Valdinoci, [A logistic equation with nonlocal interactions](#). *Kinet. Relat. Models* **10** (2017), no. 1, 141–170
- [9] E. B. Davies, [Heat kernels and spectral theory](#). Cambridge Tracts in Math. 92, Cambridge University Press, Cambridge, 1989
- [10] S. Dipierro, G. Giacomini, and E. Valdinoci, Diffusive processes modeled on the spectral fractional Laplacian with Dirichlet boundary conditions. To appear in *Analysis and Numerics of Design, Control and Inverse Problems*, Springer INdAM Ser., Springer, Cham
- [11] S. Dipierro, G. Giacomini, and E. Valdinoci, Diffusive processes modeled on the spectral fractional Laplacian with Neumann boundary conditions. To appear in *Analysis and Numerics of Design, Control and Inverse Problems*, Springer INdAM Ser., Springer, Cham
- [12] S. Dipierro, G. Giacomini, and E. Valdinoci, [Efficiency functionals for the Lévy flight foraging hypothesis](#). *J. Math. Biol.* **85** (2022), no. 4, article no. 33, 50 pp.
- [13] S. Dipierro, G. Giacomini, and E. Valdinoci, [Analysis of the Lévy flight foraging hypothesis in  \$\mathbb{R}^n\$  and unreliability of the most rewarding strategies](#). *SIAM J. Appl. Math.* **83** (2023), no. 5, 1935–1968

- [14] S. Dipierro, E. Proietti Lippi, and E. Valdinoci, [\(Non\)local logistic equations with Neumann conditions](#). *Ann. Inst. H. Poincaré C Anal. Non Linéaire* **40** (2023), no. 5, 1093–1166
- [15] S. Dipierro and E. Valdinoci, [Description of an ecological niche for a mixed local/nonlocal dispersal: an evolution equation and a new Neumann condition arising from the superposition of Brownian and Lévy processes](#). *Phys. A* **575** (2021), article no. 126052, 20 pp.
- [16] B. Dybiec, E. Gudowska-Nowak, E. Barkai, and A. A. Dubkov, [Lévy flights versus Lévy walks in bounded domains](#). *Phys. Rev. E* **95** (2017), no. 5, article no. 052102, 13 pp.
- [17] A. M. Edwards, R. A. Phillips, N. W. Watkins, M. P. Freeman, E. J. Murphy, V. Afanasyev, S. V. Buldyrev, M. G. E. da Luz, E. P. Raposo, H. E. Stanley, and G. M. Viswanathan, [Revisiting Lévy flight search patterns of wandering albatrosses, bumblebees and deer](#). *Nature* **449** (2007), 1044–1048
- [18] L. C. Evans, *Partial differential equations*. 2nd edn., Grad. Stud. Math. 19, American Mathematical Society, Providence, RI, 2010
- [19] R. Gadagkar, *Experiments in animal behaviour: cutting-edge research at trifling cost*. e-Books of the Indian Academy of Sciences, Indian Academy of Sciences, Bangalore, 2021
- [20] N. E. Humphries, N. Queiroz, J. R. M. Dyer, N. G. Pade, M. K. Musyl, K. M. Schaefer, D. W. Fuller, J. M. Brunnschweiler, T. K. Doyle, J. D. R. Houghton et al., [Environmental context explains Lévy and Brownian movement patterns of marine predators](#). *Nature* **465** (2010), no. 7301, 1066–1069
- [21] N. E. Humphries, H. Weimerskirch, N. Queiroz, E. J. Southall, and D. W. Sims, [Foraging success of biological Lévy flights recorded in situ](#). *PNAS* **109** (2012), no. 19, 7169–7174
- [22] N. E. Humphries, H. Weimerskirch, and D. W. Sims, [A new approach for objective identification of turns and steps in organism movement data relevant to random walk modelling](#). *Methods in Ecology and Evolution* **4** (2013), no. 10, 930–938
- [23] K. K. Kataria and P. Vellaisamy, [On densities of the product, quotient and power of independent subordinators](#). *J. Math. Anal. Appl.* **462** (2018), no. 2, 1627–1643
- [24] J. Klafter and I. M. Sokolov, [Anomalous diffusion spreads its wings](#). *Physics World* **18** (2005), no. 8, 1–29
- [25] N. Levernier, J. Textor, O. Bénichou, and R. Voituriez, [Inverse square Lévy walks are not optimal search strategies for  \$d \geq 2\$](#) . *Phys. Rev. Lett.* **124** (2020), no. 8, article no. 080601, 5 pp.
- [26] N. Levernier, J. Textor, O. Bénichou, and R. Voituriez, [Reply to “Comment on ‘Inverse square Lévy walks are not optimal search strategies for  \$d \geq 2\$ ’”](#). *Phys. Rev. Lett.* **126** (2021), no. 4, article no. 048902
- [27] E. Montefusco, B. Pellacci, and G. Verzini, [Fractional diffusion with Neumann boundary conditions: the logistic equation](#). *Discrete Contin. Dyn. Syst. Ser. B* **18** (2013), no. 8, 2175–2202
- [28] G. Pagnini and S. Vitali, [Should I stay or should I go? Zero-size jumps in random walks for Lévy flights](#). *Fract. Calc. Appl. Anal.* **24** (2021), no. 1, 137–167
- [29] M. H. Protter, [Can one hear the shape of a drum? revisited](#). *SIAM Rev.* **29** (1987), no. 2, 185–197

- [30] G. Ramos-Fernández, J. L. Mateos, O. Miramontes, G. Cocho, H. Larralde, and B. Ayala-Orozco, [Lévy walk patterns in the foraging movements of spider monkeys \(\*Ateles geoffroyi\*\)](#). *Behavioral Ecology and Sociobiology* **55** (2004), no. 3, 223–230
- [31] A. M. Reynolds, [Current status and future directions of Lévy walk research](#). *Biol. Open* **7** (2018), no. 1, 1–5
- [32] D. W. Sims, E. J. Southall, N. E. Humphries, G. C. Hays, C. J. A. Bradshaw, J. W. Pitchford, A. James, M. Z. Ahmed, A. S. Brierley, M. A. Hindell et al., [Scaling laws of marine predator search behaviour](#). *Nature* **451** (2008), no. 7182, 1098–1102
- [33] A. V. Skorohod, Asymptotic formulas for stable distribution laws. In *Select. Transl. Math. Statist. and Probability, Vol. 1*, pp. 157–161, American Mathematical Society, Providence, RI, 1961
- [34] R. Song and Z. Vondraček, [Potential theory of subordinate killed Brownian motion in a domain](#). *Probab. Theory Related Fields* **125** (2003), no. 4, 578–592
- [35] J. Sprekels and E. Valdinoci, [A new type of identification problems: optimizing the fractional order in a nonlocal evolution equation](#). *SIAM J. Control Optim.* **55** (2017), no. 1, 70–93
- [36] M. V. Srinivasan, S. Zhang, M. Altwein, and J. Tautz, [Honeybee navigation: nature and calibration of the “odometer”](#). *Science* **287** (2000), no. 5454, 851–853
- [37] D. W. Stevens and J. R. Krebs, *Foraging theory monographs in behavior and ecology*. Princeton University Press, New Jersey, 1986
- [38] E. Valdinoci, From the long jump random walk to the fractional Laplacian. *Bol. Soc. Esp. Mat. Apl. SeMA* (2009), no. 49, 33–44
- [39] G. M. Viswanathan, V. Afanasyev, S. V. Buldyrev, S. Havlin, M. G. E. Da Luz, E. P. Raposo, and H. E. Stanley, [Lévy flights in random searches](#). *Phys. A* **282** (2000), no. 1-2, 1–12
- [40] G. M. Viswanathan, V. Afanasyev, S. V. Buldyrev, E. J. Murphy, P. A. Prince, and H. E. Stanley, [Lévy flight search patterns of wandering albatrosses](#). *Nature* **381** (1996), 413–415
- [41] G. M. Viswanathan, F. Bartumeus, S. V. Buldyrev, J. Catalan, U. L. Fulco, S. Havlin, M. G. E. Da Luz, M. L. Lyra, E. P. Raposo, and H. E. Stanley, [Lévy flight random searches in biological phenomena](#). *Phys. A* **314** (2002), no. 1-4, 208–213
- [42] G. M. Viswanathan, S. V. Buldyrev, S. Havlin, M. G. E. Da Luz, E. P. Raposo, and H. E. Stanley, [Optimizing the success of random searches](#). *Nature* **401** (1999), no. 6756, 911–914
- [43] V. J. Wearmouth, M. J. McHugh, N. E. Humphries, A. Naegelen, M. Z. Ahmed, E. J. Southall, A. M. Reynolds, and D. W. Sims, [Scaling laws of ambush predator “waiting” behaviour are tuned to a common ecology](#). *Proc. R. Soc. B* **281** (2014), 1–9
- [44] M. Wittlinger, R. Wehner, and H. Wolf, [The ant odometer: stepping on stilts and stumps](#). *Science* **312** (2006), 1965–1967
- [45] Q. S. Zhang, [The boundary behavior of heat kernels of Dirichlet Laplacians](#). *J. Differential Equations* **182** (2002), no. 2, 416–430