References

[1] I. Abraham, Y. Bartal, and O. Neiman, Advances in metric embedding theory. *Adv. Math.* **228** (2011), no. 6, 3026–3126

[2] D. Achlioptas, Database-friendly random projections: Johnson–Lindenstrauss with binary coins. *J. Comput. System Sci.* **66** (2003), no. 4, 671–687

[3] F. Albiac and N. J. Kalton, *Topics in Banach space theory*. Grad. Texts in Math. 233, Springer, New York, 2006

[4] N. Alon, R. M. Karp, D. Peleg, and D. B. West, A graph-theoretic game and its application to the k-server problem (extended abstract). In *On-line algorithms, proceedings of a DIMACS workshop, new brunswick, new jersey, usa, february 11-13, 1991*, edited by L. A. McGeoch and D. D. Sleator, pp. 1–10, DIMACS Ser. Discrete Math. Theoret. Comput. Sci. 7, DIMACS/AMS, 1991

[5] N. Alon and V. D. Milman, Embedding of l_∞^k in finite-dimensional Banach spaces. *Israel J. Math.* **45** (1983), no. 4, 265–280

[6] N. Alon and A. Naor, Approximating the cut-norm via Grothendieck's inequality. *SIAM J. Comput.* **35** (2006), no. 4, 787–803

[7] F. Alter and V. Caselles, Uniqueness of the Cheeger set of a convex body. *Nonlinear Anal.* **70** (2009), no. 1, 32–44

[8] F. Alter, V. Caselles, and A. Chambolle, A characterization of convex calibrable sets in $\mathbb{R}^N$. *Math. Ann.* **332** (2005), no. 2, 329–366

[9] L. Ambrosio, N. Fusco, and D. Pallara, *Functions of bounded variation and free discontinuity problems*. Oxford Math. Monogr., The Clarendon Press, Oxford University Press, New York, 2000

[10] L. Ambrosio, N. Gigli, and G. Savaré, *Gradient flows in metric spaces and in the space of probability measures*. 2nd edn., Lectures Math. ETH Zürich, Birkhäuser, Basel, 2008

[11] L. Ambrosio and D. Puglisi, Linear extension operators between spaces of Lipschitz maps and optimal transport. *J. Reine Angew. Math.* **764** (2020), 1–21

[12] G. W. Anderson, A. Guionnet, and O. Zeitouni, *An introduction to random matrices*. Cambridge Stud. Adv. Math. 118, Cambridge University Press, Cambridge, 2010

[13] A. Andoni and P. Indyk, Near-optimal hashing algorithms for approximate nearest neighbor in high dimensions. In *47th annual IEEE symposium on foundations of computer science (FOCS 2006), 21-24 october 2006, berkeley, california, usa, proceedings*, pp. 459–468, IEEE Computer Society, 2006

[14] G. E. Andrews, R. Askey, and R. Roy, *Special functions*. Encyclopedia Math. Appl. 71, Cambridge University Press, Cambridge, 1999

[15] R. Anisca, A. Tcaciuc, and N. Tomczak-Jaegermann, Structure of normed spaces with extremal distance to the Euclidean space. *Houston J. Math.* **31** (2005), no. 1, 267–283

[16] R. F. Arens and J. Eells, Jr., On embedding uniform and topological spaces. *Pacific J. Math.* **6** (1956), 397–403

[17] T. Austin, A CAT(0)-valued pointwise ergodic theorem. *J. Topol. Anal.* **3** (2011), no. 2, 145–152

[18] B. Awerbuch and D. Peleg, Sparse partitions (extended abstract). In *31st annual symposium on foundations of computer science, st. louis, missouri, usa, october 22-24, 1990, volume II*, pp. 503–513, IEEE Computer Society, 1990

[19] K. Ball, Volumes of sections of cubes and related problems. In *Geometric aspects of functional analysis (1987–88)*, pp. 251–260, Lecture Notes in Math. 1376, Springer, Berlin, 1989

[20] K. Ball, Normed spaces with a weak-Gordon–Lewis property. In *Functional analysis (Austin, TX, 1987/1989)*, pp. 36–47, Lecture Notes in Math. 1470, Springer, Berlin, 1991

[21] K. Ball, Shadows of convex bodies. *Trans. Amer. Math. Soc.* **327** (1991), no. 2, 891–901

[22] K. Ball, Volume ratios and a reverse isoperimetric inequality. *J. London Math. Soc. (2)* **44** (1991), no. 2, 351–359

[23] K. Ball, Markov chains, Riesz transforms and Lipschitz maps. *Geom. Funct. Anal.* **2** (1992), no. 2, 137–172

[24] K. Ball, Convex geometry and functional analysis. In *Handbook of the geometry of Banach spaces, Vol. I*, pp. 161–194, North-Holland, Amsterdam, 2001

[25] K. Ball and A. Pajor, Convex bodies with few faces. *Proc. Amer. Math. Soc.* **110** (1990), no. 1, 225–231

[26] K. M. Ball, K. J. Böröczky, and A. Naor, Approximate isoperimetry for convex polytopes. Forthcoming manuscript

[27] S. Banach, *Théorie des opérations linéaires*. Éditions Jacques Gabay, Sceaux, 1993

[28] I. Bárány and Z. Füredi, Computing the volume is difficult. *Discrete Comput. Geom.* **2** (1987), no. 4, 319–326

[29] Y. Bartal, Probabilistic approximation of metric spaces and its algorithmic applications. In *37th Annual Symposium on Foundations of Computer Science (Burlington, VT, 1996)*, pp. 184–193, IEEE Computer Society Press, Los Alamitos, CA, 1996

[30] Y. Bartal, On approximating arbitrary metrices by tree metrics. In *STOC '98 (Dallas, TX)*, pp. 161–168, ACM, New York, 1999

[31] F. Barthe, O. Guédon, S. Mendelson, and A. Naor, A probabilistic approach to the geometry of the l_p^n-ball. *Ann. Probab.* **33** (2005), no. 2, 480–513

[32] F. Barthe and A. Naor, Hyperplane projections of the unit ball of ℓ_p^n. *Discrete Comput. Geom.* **27** (2002), no. 2, 215–226

[33] G. Basso, Fixed point theorems for metric spaces with a conical geodesic bicombing. *Ergodic Theory Dynam. Systems* **38** (2018), no. 5, 1642–1657

[34] G. Bennett, Schur multipliers. *Duke Math. J.* **44** (1977), no. 3, 603–639

[35] G. Bennett, V. Goodman, and C. M. Newman, Norms of random matrices. *Pacific J. Math.* **59** (1975), no. 2, 359–365

[36] Y. Benyamini and J. Lindenstrauss, *Geometric nonlinear functional analysis. Vol. 1*. Amer. Math. Soc. Colloq. Publ. 48, American Mathematical Society, Providence, RI, 2000

[37] R. Bhatia, *Matrix analysis*. Grad. Texts in Math. 169, Springer, New York, 1997

[38] V. Bhattiprolu, E. Lee, and A. Naor, A framework for quadratic form maximization over convex sets through nonconvex relaxations. In *STOC '21: 53rd annual ACM SIGACT symposium on theory of computing, virtual event, italy, june 21-25, 2021*, edited by S. Khuller and V. V. Williams, pp. 870–881, ACM, 2021

[39] W. Blaschke, Über affine Geometrie VII: Neue Extremeigenschaften von Ellipse und Ellipsoid. Leipz. Ber. 69, 306–318, 1917

[40] S. Bochner, Integration von Funktionen, deren Werte die Elemente eines Vektorraumes sind. *Fundam. Math.* **20** (1933), 262–276

[41] E. D. Bolker, A class of convex bodies. *Trans. Amer. Math. Soc.* **145** (1969), 323–345

[42] F. Bolley, Separability and completeness for the Wasserstein distance. In *Séminaire de probabilités XLI*, pp. 371–377, Lecture Notes in Math. 1934, Springer, Berlin, 2008

[43] J. Bourgain, A remark on finite-dimensional P_λ-spaces. *Studia Math.* **72** (1982), no. 3, 285–289

[44] J. Bourgain, On martingales transforms in finite-dimensional lattices with an appendix on the K-convexity constant. *Math. Nachr.* **119** (1984), 41–53

[45] J. Bourgain, On dimension free maximal inequalities for convex symmetric bodies in $\mathbf{R}^n$. In *Geometrical aspects of functional analysis (1985/86)*, pp. 168–176, Lecture Notes in Math. 1267, Springer, Berlin, 1987

[46] J. Bourgain, Remarks on the extension of Lipschitz maps defined on discrete sets and uniform homeomorphisms. In *Geometrical aspects of functional analysis (1985/86)*, pp. 157–167, Lecture Notes in Math. 1267, Springer, Berlin, 1987

[47] J. Bourgain and J. Lindenstrauss, Projection bodies. In *Geometric aspects of functional analysis (1986/87)*, pp. 250–270, Lecture Notes in Math. 1317, Springer, Berlin, 1988

[48] J. Bourgain, J. Lindenstrauss, and V. Milman, Approximation of zonoids by zonotopes. *Acta Math.* **162** (1989), no. 1-2, 73–141

[49] J. Bourgain and V. D. Milman, Sections euclidiennes et volume des corps symétriques convexes dans $\mathbf{R}^n$. *C. R. Acad. Sci. Paris Sér. I Math.* **300** (1985), no. 13, 435–438

[50] J. Bourgain and V. D. Milman, New volume ratio properties for convex symmetric bodies in $\mathbf{R}^n$. *Invent. Math.* **88** (1987), no. 2, 319–340

[51] J. Bourgain and S. J. Szarek, The Banach–Mazur distance to the cube and the Dvoretzky-Rogers factorization. *Israel J. Math.* **62** (1988), no. 2, 169–180

[52] J. Bourgain and L. Tzafriri, Invertibility of "large" submatrices with applications to the geometry of Banach spaces and harmonic analysis. *Israel J. Math.* **57** (1987), no. 2, 137–224

[53] L. Brasco, On principal frequencies and isoperimetric ratios in convex sets. *Ann. Fac. Sci. Toulouse Math. (6)* **29** (2020), no. 4, 977–1005

[54] M. Braverman and A. Naor, Quantitative Wasserstein rounding. Forthcoming manuscript

[55] S. Brazitikos, A. Giannopoulos, P. Valettas, and B.-H. Vritsiou, *Geometry of isotropic convex bodies*. Math. Surveys Monogr. 196, American Mathematical Society, Providence, RI, 2014

[56] E. Breuillard, M. W. Liebeck, A. Naor, and A. Rizzoli, On the inverse problem for isometry groups of Banach spaces. Forthcoming manuscript

[57] M. R. Bridson and A. Haefliger, *Metric spaces of non-positive curvature*. Grundlehren Math. Wiss. 319, Springer, Berlin, 1999

[58] B. Brinkman and M. Charikar, On the impossibility of dimension reduction in l_1. *J. ACM* **52** (2005), no. 5, 766–788

[59] R. L. Brooks, On colouring the nodes of a network. *Proc. Cambridge Philos. Soc.* **37** (1941), 194–197

[60] A. Brudnyĭ and Y. Brudnyĭ, Simultaneous extensions of Lipschitz functions. *Uspekhi Mat. Nauk* **60** (2005), no. 6(366), 53–72

[61] A. Brudnyi and Y. Brudnyi, Extension of Lipschitz functions defined on metric subspaces of homogeneous type. *Rev. Mat. Complut.* **19** (2006), no. 2, 347–359

[62] A. Brudnyi and Y. Brudnyi, Linear and nonlinear extensions of Lipschitz functions from subsets of metric spaces. *Algebra i Analiz* **19** (2007), no. 3, 106–118

[63] A. Brudnyi and Y. Brudnyi, Metric spaces with linear extensions preserving Lipschitz condition. *Amer. J. Math.* **129** (2007), no. 1, 217–314

[64] A. Brudnyi and Y. Brudnyi, *Methods of geometric analysis in extension and trace problems. Volume 2*. Monogr. Math. 103, Birkhäuser/Springer, Basel, 2012

[65] Y. Brudnyi and P. Shvartsman, Stability of the Lipschitz extension property under metric transforms. *Geom. Funct. Anal.* **12** (2002), no. 1, 73–79

[66] J. A. Brudnyĭ and N. J. Krugljak, Functors of real interpolation. *Dokl. Akad. Nauk SSSR* **256** (1981), no. 1, 14–17

[67] D. Bucur and I. Fragalà, Blaschke–Santaló and Mahler inequalities for the first eigenvalue of the Dirichlet Laplacian. *Proc. Lond. Math. Soc. (3)* **113** (2016), no. 3, 387–417

[68] D. Bucur and I. Fragalà, Reverse Faber–Krahn and Mahler inequalities for the Cheeger constant. *Proc. Roy. Soc. Edinburgh Sect. A* **148** (2018), no. 5, 913–937

[69] P. Buser, A note on the isoperimetric constant. *Ann. Sci. École Norm. Sup. (4)* **15** (1982), no. 2, 213–230

[70] G. J. Butler, Simultaneous packing and covering in euclidean space. *Proc. London Math. Soc. (3)* **25** (1972), 721–735

[71] G. Calinescu, H. Karloff, and Y. Rabani, Approximation algorithms for the 0-extension problem. *SIAM J. Comput.* **34** (2004/05), no. 2, 358–372

[72] B. Carl, Inequalities of Bernstein–Jackson-type and the degree of compactness of operators in Banach spaces. *Ann. Inst. Fourier (Grenoble)* **35** (1985), no. 3, 79–118

[73] B. Carl and A. Pajor, Gel'fand numbers of operators with values in a Hilbert space. *Invent. Math.* **94** (1988), no. 3, 479–504

[74] V. Caselles, A. Chambolle, and M. Novaga, Uniqueness of the Cheeger set of a convex body. *Pacific J. Math.* **232** (2007), no. 1, 77–90

[75] G. D. Chakerian and P. Filliman, The measures of the projections of a cube. *Studia Sci. Math. Hungar.* **21** (1986), no. 1-2, 103–110

[76] M. Charikar, C. Chekuri, A. Goel, S. Guha, and S. A. Plotkin, Approximating a finite metric by a small number of tree metrics. In *39th annual symposium on foundations of computer science, FOCS '98, november 8-11, 1998, palo alto, california, USA*, pp. 379–388, IEEE Computer Society, 1998

[77] I. Chavel, *Eigenvalues in Riemannian geometry*. Pure Appl. Math. 115, Academic Press, Orlando, FL, 1984

[78] J. Cheeger, A lower bound for the smallest eigenvalue of the Laplacian. In *Problems in analysis (Papers dedicated to Salomon Bochner, 1969)*, pp. 195–199, Princeton University Press, Princeton, NJ, 1970

[79] S. Chevet, Séries de variables aléatoires gaussiennes à valeurs dans $E\hat{\otimes}_\varepsilon F$. Application aux produits d'espaces de Wiener abstraits. In *Séminaire sur la Géométrie des Espaces de Banach (1977–1978)*, Exp. No. 19, 15 pp., École polytechnique, Palaiseau, 1978

[80] F. R. K. Chung, R. L. Graham, P. Frankl, and J. B. Shearer, Some intersection theorems for ordered sets and graphs. *J. Combin. Theory Ser. A* **43** (1986), no. 1, 23–37

[81] R. Courant and D. Hilbert, *Methods of mathematical physics. Vol. I*. Interscience Publishers, New York, NY, 1953

[82] M. Cwikel, K-divisibility of the K-functional and Calderón couples. *Ark. Mat.* **22** (1984), no. 1, 39–62

[83] M. M. Day, Polygons circumscribed about closed convex curves. *Trans. Amer. Math. Soc.* **62** (1947), 315–319

[84] A. Defant and C. Michels, Norms of tensor product identities. *Note Mat.* **25** (2005/06), no. 1, 129–166

[85] A. Defant and C. Prengel, Volume estimates in spaces of homogeneous polynomials. *Math. Z.* **261** (2009), no. 4, 909–932

[86] D. Descombes and U. Lang, Convex geodesic bicombings and hyperbolicity. *Geom. Dedicata* **177** (2015), 367–384

[87] J. Diestel, J. H. Fourie, and J. Swart, *The metric theory of tensor products*. American Mathematical Society, Providence, RI, 2008

[88] A. Doležalová and J. Vybíral, On the volume of unit balls of finite-dimensional Lorentz spaces. *J. Approx. Theory* **255** (2020), article no. 105407, 20 pp.

[89] R. Durrett, *Probability—theory and examples*. Camb. Ser. Stat. Probab. Math. 49, Cambridge University Press, Cambridge, 2019

[90] A. Dvoretzky, Some results on convex bodies and Banach spaces. In *Proc. Internat. Sympos. Linear Spaces (Jerusalem, 1960)*, pp. 123–160, Jerusalem Academic Press, Jerusalem; Pergamon, Oxford, 1961

[91] A. Dvoretzky and C. A. Rogers, Absolute and unconditional convergence in normed linear spaces. *Proc. Nat. Acad. Sci. U. S. A.* **36** (1950), 192–197

[92] J. Elton, Sign-embeddings of l_1^n. *Trans. Amer. Math. Soc.* **279** (1983), no. 1, 113–124

[93] P. Enflo, Uniform structures and square roots in topological groups. Part II. *Israel J. Math. 8 (1970), 230-252; ibid.* **8** (1970), 253–272

[94] A. Es-Sahib and H. Heinich, Barycentre canonique pour un espace métrique à courbure négative. In *Séminaire de Probabilités, XXXIII*, pp. 355–370, Lecture Notes in Math. 1709, Springer, Berlin, 1999

[95] G. Faber, Beweis, daß unter allen homogenen Membranen von gleicher Fläche und gleicher Spannung die kreisförmige den tiefsten Grundton gibt. *Münch. Ber.* **1923** (1923), 169–172

[96] J. Fakcharoenphol, S. Rao, and K. Talwar, A tight bound on approximating arbitrary metrics by tree metrics. *J. Comput. System Sci.* **69** (2004), no. 3, 485–497

[97] T. Figiel, Review of [147]. *Math. Rev.* **53** (1977), no. 3649

[98] T. Figiel and W. B. Johnson, Large subspaces of l_∞^n and estimates of the Gordon–Lewis constant. *Israel J. Math.* **37** (1980), no. 1-2, 92–112

[99] T. Figiel, J. Lindenstrauss, and V. D. Milman, The dimension of almost spherical sections of convex bodies. *Acta Math.* **139** (1977), no. 1-2, 53–94

[100] T. Figiel and N. Tomczak-Jaegermann, Projections onto Hilbertian subspaces of Banach spaces. *Israel J. Math.* **33** (1979), no. 2, 155–171

[101] A. Frieze and R. Kannan, Quick approximation to matrices and applications. *Combinatorica* **19** (1999), no. 2, 175–220

[102] R. J. Gardner, *Geometric tomography*. 2nd edn., Encyclopedia Math. Appl. 58, Cambridge University Press, New York, 2006

[103] D. J. H. Garling and Y. Gordon, Relations between some constants associated with finite dimensional Banach spaces. *Israel J. Math.* **9** (1971), 346–361

[104] A. Giannopoulos and M. Papadimitrakis, Isotropic surface area measures. *Mathematika* **46** (1999), no. 1, 1–13

[105] A. A. Giannopoulos, A note on the Banach–Mazur distance to the cube. In *Geometric aspects of functional analysis (Israel, 1992–1994)*, pp. 67–73, Oper. Theory Adv. Appl. 77, Birkhäuser, Basel, 1995

[106] A. A. Giannopoulos, A proportional Dvoretzky–Rogers factorization result. *Proc. Amer. Math. Soc.* **124** (1996), no. 1, 233–241

[107] A. A. Giannopoulos and V. D. Milman, Extremal problems and isotropic positions of convex bodies. *Israel J. Math.* **117** (2000), 29–60

[108] A. A. Giannopoulos, V. D. Milman, and M. Rudelson, Convex bodies with minimal mean width. In *Geometric aspects of functional analysis*, pp. 81–93, Lecture Notes in Math. 1745, Springer, Berlin, 2000

[109] O. Giladi, A. Naor, and G. Schechtman, Bourgain's discretization theorem. *Ann. Fac. Sci. Toulouse Math. (6)* **21** (2012), no. 4, 817–837

[110] O. Giladi, J. Prochno, C. Schütt, N. Tomczak-Jaegermann, and E. Werner, On the geometry of projective tensor products. *J. Funct. Anal.* **273** (2017), no. 2, 471–495

[111] E. D. Gluskin, The diameter of the Minkowski compactum is roughly equal to n. *Funktsional. Anal. i Prilozhen.* **15** (1981), no. 1, 72–73

[112] E. D. Gluskin, Extremal properties of orthogonal parallelepipeds and their applications to the geometry of Banach spaces. *Mat. Sb. (N.S.)* **136(178)** (1988), no. 1, 85–96

[113] G. Godefroy, A survey on Lipschitz-free Banach spaces. *Comment. Math.* **55** (2015), no. 2, 89–118

[114] G. Godefroy and N. J. Kalton, Lipschitz-free Banach spaces. *Studia Math.* **159** (2003), no. 1, 121–141

[115] Y. Gordon and M. Junge, Volume formulas in L_p-spaces. *Positivity* **1** (1997), no. 1, 7–43

[116] Y. Gordon and M. Junge, Volume ratios in L_p-spaces. *Studia Math.* **136** (1999), no. 2, 147–182

[117] Y. Gordon, M. Junge, and N. J. Nielsen, The relations between volume ratios and new concepts of GL constants. *Positivity* **1** (1997), no. 4, 359–379

[118] Y. Gordon and R. Loewy, Uniqueness of (Δ) bases and isometries of Banach spaces. *Math. Ann.* **241** (1979), no. 2, 159–180

[119] M. Gromov, Random walk in random groups. *Geom. Funct. Anal.* **13** (2003), no. 1, 73–146

[120] M. Gromov and V. D. Milman, Generalization of the spherical isoperimetric inequality to uniformly convex Banach spaces. *Compositio Math.* **62** (1987), no. 3, 263–282

[121] A. Grothendieck, Résumé de la théorie métrique des produits tensoriels topologiques. *Bol. Soc. Mat. São Paulo* **8** (1953), 1–79

[122] B. Grünbaum, Projection constants. *Trans. Amer. Math. Soc.* **95** (1960), 451–465

[123] M. A. Gunes and A. Naor, The separation modulus of unitary ideals. Forthcoming manuscript

[124] A. Gupta, R. Krauthgamer, and J. R. Lee, Bounded geometries, fractals, and low-distortion embeddings. In *44th symposium on foundations of computer science (FOCS 2003), 11-14 october 2003, cambridge, ma, usa, proceedings*, pp. 534–543, IEEE Computer Society, 2003

[125] V. I. Gurariĭ, M. Ĭ. Kadec, and V. I. Macaev, Distances between finite-dimensional analogs of the L_p-spaces. *Mat. Sb. (N.S.)* **70 (112)** (1966), 481–489

[126] H. Hadwiger, *Vorlesungen über Inhalt, Oberfläche und Isoperimetrie*. Springer, Berlin, 1957

[127] G. H. Hardy and J. E. Littlewood, Bilinear forms bounded in spaces (p, q). *Quart. J. Math. Oxford* **5** (1934), no. 1, 241–254

[128] M. Henk, Löwner–John ellipsoids. In *Optimization Stories: 21st International Symposium on Mathematical Programming. Berlin, Germany, 19–24 August 2012*, pp. 95–106, Doc. Math. Ser., European Mathematical Society, Zürich, 2012

[129] A. Henrot, *Extremum problems for eigenvalues of elliptic operators*. Front. Math., Birkhäuser, Basel, 2006

[130] A. Henrot and M. Pierre, *Shape variation and optimization*. EMS Tracts Math. 28, European Mathematical Society, Zürich, 2018

[131] T. Hytönen, S. Li, and A. Naor, Quantitative affine approximation for UMD targets. *Discrete Anal.* (2016), article no. 6, 37 pp.

[132] T. Hytönen and A. Naor, Heat flow and quantitative differentiation. *J. Eur. Math. Soc. (JEMS)* **21** (2019), no. 11, 3415–3466

[133] M. Q. Jacobs, Measurable multivalued mappings and Lusin's theorem. *Trans. Amer. Math. Soc.* **134** (1968), 471–481

[134] R. C. James, Nonreflexive spaces of type 2. *Israel J. Math.* **30** (1978), no. 1-2, 1–13

[135] R. E. Jamison and W. H. Ruckle, Factoring absolutely convergent series. *Math. Ann.* **224** (1976), no. 2, 143–148

[136] T. Jech, *Set theory*. Springer Monogr. Math., Springer, Berlin, 2003

[137] F. John, Extremum problems with inequalities as subsidiary conditions. In *Studies and Essays Presented to R. Courant on his 60th Birthday, January 8, 1948*, pp. 187–204, Interscience Publishers, New York, NY, 1948

[138] W. B. Johnson and J. Lindenstrauss, Extensions of Lipschitz mappings into a Hilbert space. In *Conference in modern analysis and probability (New Haven, Conn., 1982)*, pp. 189–206, Contemp. Math. 26, American Mathematical Society, Providence, RI, 1984

[139] W. B. Johnson and J. Lindenstrauss, Basic concepts in the geometry of Banach spaces. In *Handbook of the geometry of Banach spaces, Vol. I*, pp. 1–84, North-Holland, Amsterdam, 2001

[140] W. B. Johnson, J. Lindenstrauss, and G. Schechtman, Extensions of Lipschitz maps into Banach spaces. *Israel J. Math.* **54** (1986), no. 2, 129–138

[141] W. B. Johnson and A. Naor, The Johnson–Lindenstrauss lemma almost characterizes Hilbert space, but not quite. *Discrete Comput. Geom.* **43** (2010), no. 3, 542–553

[142] W. B. Johnson and G. Schechtman, On subspaces of L_1 with maximal distances to Euclidean space. In *Proceedings of research workshop on Banach space theory (Iowa City, Iowa, 1981)*, pp. 83–96, The University of Iowa, Iowa City, IA, 1982

[143] W. B. Johnson and G. Schechtman, Finite dimensional subspaces of L_p. In *Handbook of the geometry of Banach spaces, Vol. I*, pp. 837–870, North-Holland, Amsterdam, 2001

[144] I. Joó and L. L. Stachó, Generalization of an inequality of G. Pólya concerning the eigenfrequences of vibrating bodies. *Publ. Inst. Math. (Beograd) (N.S.)* **31(45)** (1982), 65–72

[145] Z. Kabluchko and J. Prochno, The maximum entropy principle and volumetric properties of Orlicz balls. *J. Math. Anal. Appl.* **495** (2021), no. 1, article no. 124687, 19 pp.

[146] Z. Kabluchko, J. Prochno, and C. Thäle, Exact asymptotic volume and volume ratio of Schatten unit balls. *J. Approx. Theory* **257** (2020), article no. 105457, 13 pp.

[147] M. I. Kadec and B. S. Mitjagin, Complemented subspaces in Banach spaces. *Uspehi Mat. Nauk* **28** (1973), no. 6(174), 77–94

[148] J.-P. Kahane, Sur les sommes vectorielles $\sum \pm u_n$. *C. R. Acad. Sci. Paris* **259** (1964), 2577–2580

[149] N. J. Kalton, Spaces of Lipschitz and Hölder functions and their applications. *Collect. Math.* **55** (2004), no. 2, 171–217

[150] N. J. Kalton, The complemented subspace problem revisited. *Studia Math.* **188** (2008), no. 3, 223–257

[151] N. J. Kalton, The uniform structure of Banach spaces. *Math. Ann.* **354** (2012), no. 4, 1247–1288

[152] D. Karger, R. Motwani, and M. Sudan, Approximate graph coloring by semidefinite programming. *J. ACM* **45** (1998), no. 2, 246–265

[153] B. S. Kašin, The widths of certain finite-dimensional sets and classes of smooth functions. *Izv. Akad. Nauk SSSR Ser. Mat.* **41** (1977), no. 2, 334–351, 478

[154] A. S. Kechris, *Classical descriptive set theory*. Grad. Texts in Math. 156, Springer, New York, 1995

[155] M. D. Kirszbraun, Über die zusammenziehenden und Lipschitzchen Transformationen. *Fundam. Math.* **22** (1934), 77–108

[156] P. N. Klein, S. A. Plotkin, and S. Rao, Excluded minors, network decomposition, and multicommodity flow. In *Proceedings of the twenty-fifth annual ACM symposium on theory of computing, may 16-18, 1993, san diego, ca, USA*, edited by S. R. Kosaraju, D. S. Johnson, and A. Aggarwal, pp. 682–690, ACM, 1993

[157] A. Koldobsky, D. Ryabogin, and A. Zvavitch, Projections of convex bodies and the Fourier transform. *Israel J. Math.* **139** (2004), 361–380

[158] M. Kozdoba, *Extension of Banach space valued Lipschitz functions*. Master's thesis, Technion-Israel Institute of Technology, 2005

[159] E. Krahn, Über Minimaleigenschaften der Kugel in drei und mehr Dimensionen. *Acta Univ.* **9** (1926), 1–44

[160] R. Krauthgamer, J. R. Lee, M. Mendel, and A. Naor, Measured descent: A new embedding method for finite metrics. *Geom. Funct. Anal.* **15** (2005), no. 4, 839–858

[161] K. Kuratowski and C. Ryll-Nardzewski, A general theorem on selectors. *Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys.* **13** (1965), 397–403

[162] S. Kwapień, Isomorphic characterizations of inner product spaces by orthogonal series with vector valued coefficients. *Studia Math.* **44** (1972), 583–595

[163] S. a. Kwapień and C. Schütt, Some combinatorial and probabilistic inequalities and their application to Banach space theory. II. *Studia Math.* **95** (1989), no. 2, 141–154

[164] I. Kyrezi, On the entropy of the convex hull of finite sets. *Proc. Amer. Math. Soc.* **128** (2000), no. 8, 2393–2403

[165] U. Lang, B. Pavlović, and V. Schroeder, Extensions of Lipschitz maps into Hadamard spaces. *Geom. Funct. Anal.* **10** (2000), no. 6, 1527–1553

[166] U. Lang and T. Schlichenmaier, Nagata dimension, quasisymmetric embeddings, and Lipschitz extensions. *Int. Math. Res. Not.* (2005), no. 58, 3625–3655

[167] R. Latała and K. Oleszkiewicz, Gaussian measures of dilatations of convex symmetric sets. *Ann. Probab.* **27** (1999), no. 4, 1922–1938

[168] M. Ledoux and M. Talagrand, *Probability in Banach spaces*. Ergeb. Math. Grenzgeb. 23, Springer, Berlin, 1991

[169] J. R. Lee, M. Mendel, and A. Naor, Metric structures in L_1: Dimension, snowflakes, and average distortion. *European J. Combin.* **26** (2005), no. 8, 1180–1190

[170] J. R. Lee and A. Naor, Metric decomposition, smooth measures, and clustering. 2003, unpublished manuscript, available on request

[171] J. R. Lee and A. Naor, Absolute Lipschitz extendability. *C. R. Math. Acad. Sci. Paris* **338** (2004), no. 11, 859–862

[172] J. R. Lee and A. Naor, Embedding the diamond graph in L_p and dimension reduction in L_1. *Geom. Funct. Anal.* **14** (2004), no. 4, 745–747

[173] J. R. Lee and A. Naor, Extending Lipschitz functions via random metric partitions. *Invent. Math.* **160** (2005), no. 1, 59–95

[174] L. Lefton and D. Wei, Numerical approximation of the first eigenpair of the p-Laplacian using finite elements and the penalty method. *Numer. Funct. Anal. Optim.* **18** (1997), no. 3-4, 389–399

[175] F. T. Leighton and S. Rao, An approximate max-flow min-cut theorem for uniform multicommodity flow problems with applications to approximation algorithms. In *29th annual symposium on foundations of computer science, white plains, new york, usa, 24-26 october 1988*, pp. 422–431, IEEE Computer Society, 1988

[176] D. R. Lewis, Finite dimensional subspaces of L_p. *Studia Math.* **63** (1978), no. 2, 207–212

[177] D. R. Lewis, Ellipsoids defined by Banach ideal norms. *Mathematika* **26** (1979), no. 1, 18–29

[178] Y. Lim, Contractive barycentric maps and L^1 ergodic theorems on the cone of positive definite matrices. *J. Math. Anal. Appl.* **459** (2018), no. 1, 291–306

[179] J. Lindenstrauss, On nonlinear projections in Banach spaces. *Michigan Math. J.* **11** (1964), 263–287

[180] J. Lindenstrauss and L. Tzafriri, On the complemented subspaces problem. *Israel J. Math.* **9** (1971), 263–269

[181] J. Lindenstrauss and L. Tzafriri, *Classical Banach spaces. I*. Springer, Berlin, 1977

[182] J. Lindenstrauss and L. Tzafriri, *Classical Banach spaces. II*. Ergeb. Math. Grenzgeb. 97, Springer, Berlin, 1979

[183] N. Linial, S. Mendelson, G. Schechtman, and A. Shraibman, Complexity measures of sign matrices. *Combinatorica* **27** (2007), no. 4, 439–463

[184] N. Linial and M. E. Saks, Decomposing graphs into regions of small diameter. In *Proceedings of the second annual ACM/SIGACT-SIAM symposium on discrete algorithms, 28-30 january 1991, san francisco, california.*, edited by A. Aggarwal, pp. 320–330, ACM/SIAM, 1991

[185] L. H. Loomis and H. Whitney, An inequality related to the isoperimetric inequality. *Bull. Amer. Math. Soc* **55** (1949), 961–962

[186] G. J. Lozanovskiĭ, Certain Banach lattices. *Sibirsk. Mat. Ž.* **10** (1969), 584–599

[187] M. Ludwig, Projection bodies and valuations. *Adv. Math.* **172** (2002), no. 2, 158–168

[188] M. Ludwig, Minkowski valuations. *Trans. Amer. Math. Soc.* **357** (2005), no. 10, 4191–4213

[189] N. Lusin, *Leçons sur les ensembles analytiques et leurs applications*. Chelsea, New York, 1972

[190] E. Lutwak, Selected affine isoperimetric inequalities. In *Handbook of convex geometry, Vol. A, B*, pp. 151–176, North-Holland, Amsterdam, 1993

[191] J. Luukkainen and E. Saksman, Every complete doubling metric space carries a doubling measure. *Proc. Amer. Math. Soc.* **126** (1998), no. 2, 531–534

[192] N. N. Luzin, Sur la classification de M. Baire. *C. R. Math. Acad. Sci. Paris* **164** (1917), 91–94

[193] K. Mahler, Ein Übertragungsprinzip für konvexe Körper. *Časopis Pěst. Mat. Fys.* **68** (1939), 93–102

[194] E. Makai, Jr. and H. Martini, The cross-section body, plane sections of convex bodies and approximation of convex bodies. I. *Geom. Dedicata* **63** (1996), no. 3, 267–296

[195] K. Makarychev and Y. Makarychev, Metric extension operators, vertex sparsifiers and Lipschitz extendability. *Israel J. Math.* **212** (2016), no. 2, 913–959

[196] P. Mankiewicz and N. Tomczak-Jaegermann, Quotients of finite-dimensional Banach spaces; random phenomena. In *Handbook of the geometry of Banach spaces, Vol. 2*, pp. 1201–1246, North-Holland, Amsterdam, 2003

[197] M. B. Marcus and G. Pisier, Characterizations of almost surely continuous p-stable random Fourier series and strongly stationary processes. *Acta Math.* **152** (1984), no. 3-4, 245–301

[198] E. Markessinis, G. Paouris, and C. Saroglou, Comparing the M-position with some classical positions of convex bodies. *Math. Proc. Cambridge Philos. Soc.* **152** (2012), no. 1, 131–152

[199] J. Matoušek, Extension of Lipschitz mappings on metric trees. *Comment. Math. Univ. Carolin.* **31** (1990), no. 1, 99–104

[200] J. Matoušek, On the distortion required for embedding finite metric spaces into normed spaces. *Israel J. Math.* **93** (1996), 333–344

[201] J. Matoušek, *Lectures on discrete geometry*. Grad. Texts in Math. 212, Springer, New York, 2002

[202] J. Matoušek, A. Nikolov, and K. Talwar, Factorization norms and hereditary discrepancy. *Int. Math. Res. Not. IMRN* **2020** (2020), no. 3, 751–780

[203] B. Maurey, Type, cotype and K-convexity. In *Handbook of the geometry of Banach spaces, Vol. 2*, pp. 1299–1332, North-Holland, Amsterdam, 2003

[204] B. Maurey and G. Pisier, Séries de variables aléatoires vectorielles indépendantes et propriétés géométriques des espaces de Banach. *Studia Math.* **58** (1976), no. 1, 45–90

[205] S. Mazur, Une remarque sur l'homéomorphie des champs fonctionels. *Studia Math.* **1** (1929), 83–85

[206] E. J. McShane, Extension of range of functions. *Bull. Amer. Math. Soc.* **40** (1934), no. 12, 837–842

[207] M. Mendel and A. Naor, Formulae for absolute extendability operators. Forthcoming manuscript

[208] M. Mendel and A. Naor, Ramsey partitions and proximity data structures. *J. Eur. Math. Soc. (JEMS)* **9** (2007), no. 2, 253–275

[209] M. Mendel and A. Naor, Metric cotype. *Ann. of Math. (2)* **168** (2008), no. 1, 247–298

[210] M. Mendel and A. Naor, Spectral calculus and Lipschitz extension for barycentric metric spaces. *Anal. Geom. Metr. Spaces* **1** (2013), 163–199

[211] M. Mendel and A. Naor, Expanders with respect to Hadamard spaces and random graphs. *Duke Math. J.* **164** (2015), no. 8, 1471–1548

[212] M. Mendel, A. Naor, and Y. Rabani, A weighted Sobolev embedding on the discrete torus and nonexistence of gentle partitions of unity. Forthcoming manuscript

[213] S. Mendelson and R. Vershynin, Entropy and the combinatorial dimension. *Invent. Math.* **152** (2003), no. 1, 37–55

[214] C. Meyer, *Matrix analysis and applied linear algebra*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2000

[215] V. D. Milman, A new proof of A. Dvoretzky's theorem on cross-sections of convex bodies. *Funkcional. Anal. i Priložen.* **5** (1971), no. 4, 28–37

[216] V. D. Milman, Almost Euclidean quotient spaces of subspaces of a finite-dimensional normed space. *Proc. Amer. Math. Soc.* **94** (1985), no. 3, 445–449

[217] V. D. Milman, Some remarks on Urysohn's inequality and volume ratio of cotype 2-spaces. In *Geometrical aspects of functional analysis (1985/86)*, pp. 75–81, Lecture Notes in Math. 1267, Springer, Berlin, 1987

[218] V. D. Milman and A. Pajor, Isotropic position and inertia ellipsoids and zonoids of the unit ball of a normed n-dimensional space. In *Geometric aspects of functional analysis (1987–88)*, pp. 64–104, Lecture Notes in Math. 1376, Springer, Berlin, 1989

[219] V. D. Milman and G. Pisier, Banach spaces with a weak cotype 2 property. *Israel J. Math.* **54** (1986), no. 2, 139–158

[220] V. D. Milman and G. Schechtman, *Asymptotic theory of finite-dimensional normed spaces*. Lecture Notes in Math. 1200, Springer, Berlin, 1986

[221] V. D. Mil'man and H. Wolfson, Minkowski spaces with extremal distance from the Euclidean space. *Israel J. Math.* **29** (1978), no. 2-3, 113–131

[222] L. Mirsky, Symmetric gauge functions and unitarily invariant norms. *Quart. J. Math. Oxford Ser. (2)* **11** (1960), 50–59

[223] D. Müller, A geometric bound for maximal functions associated to convex bodies. *Pacific J. Math.* **142** (1990), no. 2, 297–312

[224] A. Naor, A phase transition phenomenon between the isometric and isomorphic extension problems for Hölder functions between L_p spaces. *Mathematika* **48** (2001), no. 1-2, 253–271 (2003)

[225] A. Naor, The surface measure and cone measure on the sphere of l_p^n. *Trans. Amer. Math. Soc.* **359** (2007), no. 3, 1045–1079

[226] A. Naor, Class notes on Lipschitz extension from finite subsets. 2015, https://web.math.princeton.edu/∼naor/homepage%20files/extension-from-finite.pdf, visited on 29 March 2024

[227] A. Naor, Probabilistic clustering of high dimensional norms. In *Proceedings of the twenty-eighth annual ACM-SIAM symposium on discrete algorithms, SODA 2017*, pp. 690–709, 2017

[228] A. Naor, A spectral gap precludes low-dimensional embeddings. In *33rd International Symposium on Computational Geometry*, pp. article no. 50, 16 pp., LIPIcs. Leibniz Int. Proc. Inform. 77, Schloss Dagstuhl. Leibniz-Zent. Inform., Wadern, 2017

[229] A. Naor, Metric dimension reduction: A snapshot of the Ribe program. In *Proceedings of the International Congress of Mathematicians—Rio de Janeiro 2018. Vol. I. Plenary lectures*, pp. 759–837, World Scientific, Hackensack, NJ, 2018

[230] A. Naor, An average John theorem. *Geom. Topol.* **25** (2021), no. 4, 1631–1717

[231] A. Naor, Impossibility of almost extension. *Adv. Math.* **384** (2021), article no. 107761, 34 pp.

[232] A. Naor, G. Pisier, and G. Schechtman, Impossibility of dimension reduction in the nuclear norm. *Discrete Comput. Geom.* **63** (2020), no. 2, 319–345

[233] A. Naor and Y. Rabani, On Lipschitz extension from finite subsets. *Israel J. Math.* **219** (2017), no. 1, 115–161

[234] A. Naor and G. Schechtman, Lipschitz almost-extension and nonexistence of uniform embeddings of balls in Schatten classes. Forthcoming manuscript

[235] A. Naor and G. Schechtman, Obstructions to metric embeddings of Schatten classes. Forthcoming manuscript

[236] A. Naor and G. Schechtman, Planar earthmover is not in L_1. *SIAM J. Comput.* **37** (2007), no. 3, 804–826

[237] A. Naor and G. Schechtman, Pythagorean powers of hypercubes. *Ann. Inst. Fourier (Grenoble)* **66** (2016), no. 3, 1093–1116

[238] A. Naor and T. Tao, Random martingales and localization of maximal inequalities. *J. Funct. Anal.* **259** (2010), no. 3, 731–779

[239] A. Naor and T. Tao, Scale-oblivious metric fragmentation and the nonlinear Dvoretzky theorem. *Israel J. Math.* **192** (2012), no. 1, 489–504

[240] A. Naor and R. Young, Foliated corona decompositions. *Acta Math.* **229** (2022), no. 1, 55–200

[241] A. Navas, An L^1 ergodic theorem with values in a non-positively curved space via a canonical barycenter map. *Ergodic Theory Dynam. Systems* **33** (2013), no. 2, 609–623

[242] I. Newman and Y. Rabinovich, A lower bound on the distortion of embedding planar metrics into Euclidean space. *Discrete Comput. Geom.* **29** (2003), no. 1, 77–81

[243] S.-i. Ohta, Extending Lipschitz and Hölder maps between metric spaces. *Positivity* **13** (2009), no. 2, 407–425

[244] M. I. Ostrovskii, *Metric embeddings. Bilipschitz and coarse embeddings into Banach spaces*. De Gruyter Stud. Math. 49, De Gruyter, Berlin, 2013

[245] A. Pajor, Plongement de l_1^n dans les espaces de Banach complexes. *C. R. Acad. Sci. Paris Sér. I Math.* **296** (1983), no. 17, 741–743

[246] E. Parini, Reverse Cheeger inequality for planar convex sets. *J. Convex Anal.* **24** (2017), no. 1, 107–122

[247] A. Pełczyński, Linear extensions, linear averagings, and their applications to linear topological classification of spaces of continuous functions. *Dissertationes Math. Rozprawy Mat.* **58** (1968), 92 pp.

[248] D. Peleg and E. Reshef, Deterministic polylog approximation for minimum communication spanning trees (extended abstract). In *Automata, languages and programming (Aalborg, 1998)*, pp. 670–681, Lecture Notes in Comput. Sci. 1443, Springer, Berlin, 1998

[249] B. J. Pettis, Linear functionals and completely additive set functions. *Duke Math. J.* **4** (1938), no. 3, 552–565

[250] C. M. Petty, Surface area of a convex body under affine transformations. *Proc. Amer. Math. Soc.* **12** (1961), 824–828

[251] C. M. Petty, Projection bodies. In *Proc. Colloquium on Convexity (Copenhagen, 1965)*, pp. 234–241, Kobenhavns Univ. Mat. Inst., Copenhagen, 1967

[252] C. M. Petty, Isoperimetric problems. In *Proceedings of the Conference on Convexity and Combinatorial Geometry (Univ. Oklahoma, Norman, Okla., 1971)*, pp. 26–41, Department of Mathematics, The University of Oklahoma, Norman, Okla., 1971

[253] A. Pietsch and J. Wenzel, *Orthonormal systems and Banach space geometry*. Encyclopedia Math. Appl. 70, Cambridge University Press, Cambridge, 1998

[254] G. Pisier, Sur les espaces de Banach qui ne contiennent pas uniformément de l_n^1. *C. R. Acad. Sci. Paris Sér. A-B* **277** (1973), A991–A994

[255] G. Pisier, Sur les espaces de Banach de dimension finie à distance extrémale d'un espace euclidien [d'après V. D. Milman et H. Wolfson]. In *Séminaire d'Analyse Fonctionnelle (1978–1979)*, Exp. No. 16, 10 pp., École polytechnique, Palaiseau, 1979

[256] G. Pisier, Sur les espaces de Banach K-convexes. In *Seminar on Functional Analysis, 1979–1980 (French)*, Exp. No. 11, 15 pp., École polytechnique, Palaiseau, 1980

[257] G. Pisier, Un théorème sur les opérateurs linéaires entre espaces de Banach qui se factorisent par un espace de Hilbert. *Ann. Sci. École Norm. Sup. (4)* **13** (1980), no. 1, 23–43

[258] G. Pisier, Remarques sur un résultat non publié de B. Maurey. In *Seminar on Functional Analysis, 1980–1981*, Exp. No. V, 13 pp., École polytechnique, Palaiseau, 1981

[259] G. Pisier, Holomorphic semigroups and the geometry of Banach spaces. *Ann. of Math. (2)* **115** (1982), no. 2, 375–392

[260] G. Pisier, On the dimension of the l_p^n-subspaces of Banach spaces, for $1 \leqslant p < 2$. *Trans. Amer. Math. Soc.* **276** (1983), no. 1, 201–211

[261] G. Pisier, *Factorization of linear operators and geometry of Banach spaces*. CBMS Reg. Conf. Ser. Math. 60, Published for the Conference Board of the Mathematical Sciences, Washington, DC; by the American Mathematical Society, Providence, RI, 1986

[262] G. Pisier, Weak Hilbert spaces. *Proc. London Math. Soc. (3)* **56** (1988), no. 3, 547–579

[263] G. Pisier, *The volume of convex bodies and Banach space geometry*. Cambridge Tracts in Math. 94, Cambridge University Press, Cambridge, 1989

[264] G. Pisier, Dvoretzky's theorem for operator spaces. *Houston J. Math.* **22** (1996), no. 2, 399–416

[265] G. Pólya and G. Szegö, *Isoperimetric Inequalities in Mathematical Physics*. Ann. of Math. Stud. 27, Princeton University Press, Princeton, NJ, 1951

[266] S. T. Rachev and L. Rüschendorf, Approximate independence of distributions on spheres and their stability properties. *Ann. Probab.* **19** (1991), no. 3, 1311–1337

[267] H. Rademacher, Über partielle und totale differenzierbarkeit von Funktionen mehrerer Variabeln und über die Transformation der Doppelintegrale. *Math. Ann.* **79** (1919), no. 4, 340–359

[268] M. M. Rao and Z. D. Ren, *Theory of Orlicz spaces*. Monogr. Textbooks Pure Appl. Math. 146, Marcel Dekker, New York, 1991

[269] S. Rao, Small distortion and volume preserving embeddings for planar and Euclidean metrics. In *Proceedings of the Fifteenth Annual Symposium on Computational Geometry (Miami Beach, FL, 1999)*, pp. 300–306, ACM, New York, 1999

[270] O. Regev, Entropy-based bounds on dimension reduction in L_1. *Israel J. Math.* **195** (2013), no. 2, 825–832

[271] S. Reisner, Zonoids with minimal volume-product. *Math. Z.* **192** (1986), no. 3, 339–346

[272] M. Riesz, Sur les maxima des formes bilinéaires et sur les fonctionnelles linéaires. *Acta Math.* **49** (1927), no. 3-4, 465–497

[273] C. A. Rogers, A note on coverings and packings. *J. London Math. Soc.* **25** (1950), 327–331

[274] M. Rudelson and R. Vershynin, Combinatorics of random processes and sections of convex bodies. *Ann. of Math. (2)* **164** (2006), no. 2, 603–648

[275] D. Rutovitz, Some parameters associated with finite-dimensional Banach spaces. *J. London Math. Soc.* **40** (1965), 241–255

[276] R. A. Ryan, *Introduction to tensor products of Banach spaces*. Springer Monogr. Math., Springer, London, 2002

[277] J. Saint-Raymond, Le volume des idéaux d'opérateurs classiques. *Studia Math.* **80** (1984), no. 1, 63–75

[278] L. A. Santaló, An affine invariant for convex bodies of n-dimensional space. *Portugal. Math.* **8** (1949), 155–161

[279] G. Schechtman and J. Zinn, On the volume of the intersection of two L_p^n balls. *Proc. Amer. Math. Soc.* **110** (1990), no. 1, 217–224

[280] M. Schmuckenschläger, The distribution function of the convolution square of a convex symmetric body in $\mathbf{R}^n$. *Israel J. Math.* **78** (1992), no. 2-3, 309–334

[281] M. Schmuckenschläger, Petty's projection inequality and Santalo's affine isoperimetric inequality. *Geom. Dedicata* **57** (1995), no. 3, 285–295

[282] R. Schneider, *Convex bodies: the Brunn–Minkowski theory*. expanded edn., Encyclopedia Math. Appl. 151, Cambridge University Press, Cambridge, 2014

[283] R. Schneider and W. Weil, Zonoids and related topics. In *Convexity and its applications*, pp. 296–317, Birkhäuser, Basel, 1983

[284] C. Schütt, Unconditionality in tensor products. *Israel J. Math.* **31** (1978), no. 3-4, 209–216

[285] C. Schütt, On the volume of unit balls in Banach spaces. *Compositio Math.* **47** (1982), no. 3, 393–407

[286] C. Schütt, The isoperimetric quotient and some classical Banach spaces. *Israel J. Math.* **67** (1989), no. 1, 43–61

[287] G. C. Shephard, Shadow systems of convex sets. *Israel J. Math.* **2** (1964), 229–236

[288] E. Silverman, Lower semicontinuity of parametric integrals. *Trans. Amer. Math. Soc.* **175** (1973), 499–508

[289] B. Simon, *Trace ideals and their applications*. London Math. Soc. Lecture Note Ser. 35, Cambridge University Press, Cambridge, 1979

[290] A. Sobczyk, Projections in Minkowski and Banach spaces. *Duke Math. J.* **8** (1941), 78–106

[291] S. M. Srivastava, *A course on Borel sets*. Grad. Texts in Math. 180, Springer, New York, 1998

[292] K.-T. Sturm, Probability measures on metric spaces of nonpositive curvature. In *Heat kernels and analysis on manifolds, graphs, and metric spaces (Paris, 2002)*, pp. 357–390, Contemp. Math. 338, American Mathematical Society, Providence, RI, 2003

[293] S. a. Szarek and N. Tomczak-Jaegermann, On nearly Euclidean decomposition for some classes of Banach spaces. *Compositio Math.* **40** (1980), no. 3, 367–385

[294] S. a. J. Szarek, On Kashin's almost Euclidean orthogonal decomposition of l_n^1. *Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys.* **26** (1978), no. 8, 691–694

[295] S. a. J. Szarek, Spaces with large distance to l_∞^n and random matrices. *Amer. J. Math.* **112** (1990), no. 6, 899–942

[296] S. J. Szarek, On the geometry of the Banach–Mazur compactum. In *Functional analysis (Austin, TX, 1987/1989)*, pp. 48–59, Lecture Notes in Math. 1470, Springer, Berlin, 1991

[297] S. J. Szarek and M. Talagrand, An "isomorphic" version of the Sauer–Shelah lemma and the Banach–Mazur distance to the cube. In *Geometric aspects of functional analysis (1987–88)*, pp. 105–112, Lecture Notes in Math. 1376, Springer, Berlin, 1989

[298] M. Talagrand, Type, infratype and the Elton–Pajor theorem. *Invent. Math.* **107** (1992), no. 1, 41–59

[299] M. Talagrand, Embedding of l_k^∞ and a theorem of Alon and Milman. In *Geometric aspects of functional analysis (Israel, 1992–1994)*, pp. 289–293, Oper. Theory Adv. Appl. 77, Birkhäuser, Basel, 1995

[300] A. E. Taylor, A geometric theorem and its application to biorthogonal systems. *Bull. Amer. Math. Soc.* **53** (1947), 614–616

[301] G. O. Thorin, Convexity theorems generalizing those of M. Riesz and Hadamard with some applications. *Comm. Sem. Math. Univ. Lund [Medd. Lunds Univ. Mat. Sem.]* **9** (1948), 1–58

[302] K. Tikhomirov, On the Banach–Mazur distance to cross-polytope. *Adv. Math.* **345** (2019), 598–617

[303] N. Tomczak-Jaegermann, Computing 2-summing norm with few vectors. *Ark. Mat.* **17** (1979), no. 2, 273–277

[304] N. Tomczak-Jaegermann, Finite-dimensional subspaces of uniformly convex and uniformly smooth Banach lattices and trace classes S_p. *Studia Math.* **66** (1979/80), no. 3, 261–281

[305] N. Tomczak-Jaegermann, *Banach–Mazur distances and finite-dimensional operator ideals*. Pitman Monogr. Surveys Pure Appl. Math. 38, Longman Scientific & Technical, Harlow; copublished in the United States with John Wiley & Sons, New York, 1989

[306] F. Tricomi, Sulle funzioni di Bellel di ordine e argomento pressochè uguali. *Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur.* **83** (1949), 3–20

[307] C. Villani, *Optimal transport*. Grundlehren Math. Wiss. 338, Springer, Berlin, 2009

[308] A. L. Vol'berg and S. V. Konyagin, On measures with the doubling condition. *Izv. Akad. Nauk SSSR Ser. Mat.* **51** (1987), no. 3, 666–675

[309] D. H. Wagner, Survey of measurable selection theorems. *SIAM J. Control Optimization* **15** (1977), no. 5, 859–903

[310] N. Weaver, *Lipschitz algebras*. World Scientific, River Edge, NJ, 1999

[311] H. Weyl, *The Classical Groups. Their Invariants and Representations*. Princeton University Press, Princeton, NJ, 1939

[312] H. Whitney, Analytic extensions of differentiable functions defined in closed sets. *Trans. Amer. Math. Soc.* **36** (1934), no. 1, 63–89

[313] E. T. Whittaker and G. N. Watson, *A course of modern analysis. An introduction to the general theory of infinite processes and of analytic functions: with an account of the principal transcendental functions*. Cambridge University Press, New York, 1962

[314] P. Wojtaszczyk, *Banach spaces for analysts*. Cambridge Stud. Adv. Math. 25, Cambridge University Press, Cambridge, 1991

[315] C. Zong, From deep holes to free planes. *Bull. Amer. Math. Soc. (N.S.)* **39** (2002), no. 4, 533–555

Added in proof. Since the initial posting of this work, the following progress was made on some of the issues that are discussed herein. Conjecture 10 is proved in the forthcoming article [26] in the special case when $K \subseteq \mathbb{R}^n$ is an origin-symmetric convex polytope that has $O(n)$ faces. The forthcoming article [123] proves Conjecture 10 when $K \subseteq \mathsf{M}_n(\mathbb{R})$ is the unit ball of the unitary ideal $\mathsf{S}_{\mathbf{E}}$ of any 1-symmetric normed space $\mathbf{E} = (\mathbb{R}^n, \|\cdot\|_{\mathbf{E}})$; consequently, Lemma 54 and Proposition 55 hold with all of the logarithmic factors that appear in them replaced by universal constants. The forthcoming work [54] builds on the results herein while adding multiple innovations and ideas to obtain several new results. These include the estimate $\mathsf{e}(\mathbf{X}) \gtrsim \sqrt[4]{\dim(\mathbf{X})}$ for every normed space $\mathbf{X}$, which is an improvement over the value of the universal constant c that we obtained in the proof of Theorem 1. Conjecture 134 is resolved (negatively) in [54], where it is proved that $\mathsf{e}_{\mathrm{conv}}(\mathfrak{M}) \lesssim \mathsf{e}(\mathfrak{M})^2$ for every Polish metric space $(\mathfrak{M}, d_{\mathfrak{M}})$. It is also proved in [54] that $\mathsf{e}(\ell_2^n, \mathfrak{N}) \asymp \sqrt[4]{n}$ for any 1-net $\mathfrak{N}$ of ℓ_2^n and $\mathsf{e}(\ell_2^n, \mathbb{Z}^n) \asymp \sqrt[6]{n}$; both of these asymptotic evaluations of Lipschitz extension moduli answer questions that were posed in the precursor [227] of the present work. Finally, the lower order factor in the main result of [231] is removed in [54], thus showing that an old Lipschitz almost-extension result of Bourgain [46] is sharp up to universal constant factors. Beyond the aforementioned examples of statements from [54], multiple other new results on Lipschitz extension and separation moduli are obtained in [54]. The discussion in Remark 41 (more generally, the role that canonically positioned norms play herein), evolved (very substantially) to the forthcoming work [56] which investigates the question of when is it possible to construct a norm with prescribed group of isometries; as demonstrated in [56], it turns out that the answer to this old inverse problem is quite subtle.