Abstract

We review several properties of integrals of the Wigner distribution on subsets of the phase space. Along our way, we provide a theoretical proof of the invalidity of Flandrin's conjecture, a fact already proven via numerical arguments in our joint paper [J. Fourier Anal. Appl. **26** (2020), no. 1, article no. 6] with B. Delourme and T. Duyckaerts. We use also the J. G. Wood and A. J. Bracken paper [J. Math. Phys. **46** (2005), no. 4, article no. 042103], for which we offer a mathematical perspective. We review thoroughly the case of subsets of the plane whose boundary is a conic curve and show that Mehler's formula can be helpful in the analysis of these cases, including for the higher-dimensional case investigated in the paper [J. Math. Phys. **51** (2010), no. 10, article no. 102101] by E. Lieb and Y. Ostrover. Using the Feichtinger algebra, we show that, generically in the Baire sense, the Wigner distribution of a pulse in $L^2(\mathbb{R}^n)$ does not belong to $L^1(\mathbb{R}^{2n})$, providing as a byproduct a large class of examples of subsets of the phase space $\mathbb{R}^{2n}$ on which the integral of the Wigner distribution is infinite. We study as well the case of convex polygons of the plane, with a rather weak estimate depending on the number of vertices, but independent of the area of the polygon.

Keywords. Wigner distribution, signal theory, pseudo-differential operators

Mathematics Subject Classification (2020). Primary 81S30; Secondary 35P05, 42B20, 47G10, 47G30, 47N70, 94A12

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