Chapter 6

Overview of the proofs

The rest of this work is devoted to proving the main results. Since these involve a number of technical steps, we now give a brief overview of how these proofs proceed.

We commence in Chapter 7 by developing compressed sensing theory for Hilbert-valued vectors. We introduced the so-called *weighted robust Null Space Property (rNSP) over $\mathcal{V}$*, and then show in Lemma 7.4 that it implies certain error bounds for inexact minimizers of the Hilbert-valued, weighted SR-LASSO problem. Next, we introduced the *weighted Restricted Isometry Property (RIP)* and then in Lemma 7.6 we show that this property over $\mathbb{C}$ implies the weighted rNSP over $\mathcal{V}$.

In Chapter 8 we focus on the polynomial approximation problem. We first give a sufficient condition in terms of m for the measurement matrix (4.3) to satisfy the weighted RIP with high probability (Lemma 8.1). Next, we state and prove three general results (Theorems 8.2–8.4) that give error bounds for polynomial approximations obtained as inexact minimizers of the Hilbert-valued, weighted SR-LASSO problem. These results are split into the three cases considered in our main results, i.e., the algebraic and finite-dimensional case, the algebraic and infinite-dimensional case, and the exponential case. The error bounds in these results split into terms corresponding to the polynomial approximation error, the physical discretization error, the sampling error, and the error in the objective function at the inexact minimizer.

With this in mind, in Chapter 9, we first present error bounds for inexact minimizers obtained by finitely many iterations of the primal-dual iteration. See Lemma 9.2. Having done this, we then have the ingredients needed to derive the restarting scheme. We derive this scheme and present an error bound for it in Theorem 9.4.

We conclude with in Chapter 10 with the final arguments. We use the three key theorems (Theorems 8.2–8.4) and then proceed to estimate each of the aforementioned error terms. For the polynomial approximation error we appeal to several results that are given in Appendix A. For the error in the objective function we use the results shown in Chapter 9. After straightforwardly bounding the other two error terms, we finally obtain the main results.