

# Contents

<b>Overview</b>	1
<b>1 Introduction to statistical mechanics</b>	12
1.1 Gibbs measures	12
1.2 The Ising model	15
1.3 The Curie–Weiss model	20
<b>2 Convex analysis and large deviation principles</b>	22
2.1 Convex analysis	22
2.2 Large deviation principles	40
2.3 Analyzing the Curie–Weiss model	48
2.4 The envelope theorem and the Curie–Weiss magnetization	52
<b>3 Hamilton–Jacobi equations</b>	58
3.1 A Hamilton–Jacobi approach to Curie–Weiss	58
3.2 Viscosity solutions to Hamilton–Jacobi equations	63
3.3 Uniqueness of solutions via the comparison principle	67
3.4 Variational representations of viscosity solutions	74
3.5 Variational representations in the absence of convexity	86
3.6 Leveraging convexity to identify viscosity solutions	92
<b>4 Statistical inference</b>	103
4.1 From statistical inference to statistical mechanics	104
4.2 Gaussian integration by parts and concentration inequalities	109
4.3 A Hamilton–Jacobi approach to rank-one matrix estimation	121
4.4 Comparison with a concrete algorithm	141
4.5 The community detection problem	148
<b>5 Poisson point processes and extreme values</b>	157
5.1 The space of point measures	157
5.2 Point processes	163
5.3 Poisson point processes	166
5.4 Extremes of i.i.d. random variables	171
5.5 Poisson–Dirichlet processes	179
5.6 Poisson–Dirichlet cascades	186
5.7 Ultrametricity as a universal property	198

<b>6 Mean-field spin glasses . . . . .</b>	208
6.1 The Sherrington–Kirkpatrick model and the Parisi formula . . . . .	209
6.2 A first attempt at a Hamilton–Jacobi approach to the SK model . . . . .	214
6.3 Some insight from the random energy model . . . . .	217
6.4 A Hamilton–Jacobi approach to the SK model . . . . .	219
6.5 Connecting the Hamilton–Jacobi approach with the Parisi formula . . . . .	229
6.6 Towards non-convex models . . . . .	243
<b>A Basic results in analysis and probability . . . . .</b>	253
A.1 Constructing measures . . . . .	253
A.2 The Riesz representation theorem . . . . .	262
A.3 The Stone–Weierstrass theorem . . . . .	266
A.4 The Lebesgue differentiation theorem . . . . .	269
A.5 A topological characterization of weak convergence . . . . .	273
A.6 Weak convergence through tightness and uniqueness . . . . .	276
<b>S Solutions to exercises . . . . .</b>	283
S.1 Introduction to statistical mechanics . . . . .	283
S.2 Convex analysis and large deviation principles . . . . .	287
S.3 Hamilton–Jacobi equations . . . . .	301
S.4 Statistical inference . . . . .	307
S.5 Poisson point processes and extreme values . . . . .	316
S.6 Mean-field spin glasses . . . . .	328
S.7 Basic results in analysis and probability . . . . .	335
<b>References . . . . .</b>	343
<b>Index . . . . .</b>	358