

# Contents

<b>I</b>	<b>Formal derivations and macroscopic weak stability</b>	1
<b>1</b>	<b>The Vlasov–Maxwell–Boltzmann system</b>	3
1.1	The Boltzmann collision operator	5
1.2	Formal macroscopic properties	8
1.3	The mathematical framework	12
<b>2</b>	<b>Scalings and formal limits</b>	15
2.1	Incompressible viscous regimes	15
2.2	Scalings for the electromagnetic field	17
2.3	Formal analysis of the one-species asymptotics	21
2.3.1	Thermodynamic equilibrium	22
2.3.2	Macroscopic constraints	25
2.3.3	Evolution equations	27
2.3.4	Summary	33
2.3.5	The Vlasov–Poisson–Boltzmann system	36
2.4	Formal analysis of the two-species asymptotics	37
2.4.1	Thermodynamic equilibrium	39
2.4.2	The case of very weak interspecies collisions	46
2.4.3	Macroscopic hydrodynamic constraints	55
2.4.4	Hydrodynamic evolution equations	56
2.4.5	Macroscopic electrodynamic constraints and evolution	60
2.4.6	Summary	65
2.4.7	The two-species Vlasov–Poisson–Boltzmann system	69
<b>3</b>	<b>Weak stability of the limiting macroscopic systems</b>	73
3.1	The incompressible quasi-static Navier–Stokes–Fourier–Maxwell– Poisson system	74
3.2	The two-fluid incompressible Navier–Stokes–Fourier–Maxwell system with (solenoidal) Ohm’s law	77
3.2.1	Large global solutions in two dimensions	81
3.2.2	Small global solutions in three dimensions	83
3.2.3	Weak-strong stability and dissipative solutions	84
3.2.3.1	The incompressible Navier–Stokes–Maxwell system	85
3.2.3.2	The two-fluid incompressible Navier–Stokes– Maxwell system with Ohm’s law	92
3.2.3.3	The two-fluid incompressible Navier–Stokes– Maxwell system with solenoidal Ohm’s law	105

<b>II</b>	<b>Conditional convergence results</b>	119
<b>4</b>	<b>Two typical regimes</b>	121
4.1	Renormalized solutions	122
4.1.1	The Vlasov–Boltzmann equation	122
4.1.2	Coupling the Boltzmann equation with Maxwell’s equations	131
4.1.3	The setting of our conditional study	132
4.1.4	Macroscopic conservation laws	134
4.2	The incompressible quasi-static Navier–Stokes–Fourier–Maxwell–Poisson system	140
4.3	The two-fluid incompressible Navier–Stokes–Fourier–Maxwell system with (solenoidal) Ohm’s law	145
4.3.1	Weak interactions	147
4.3.2	Strong interactions	151
4.4	Outline of proofs	155
<b>5</b>	<b>Weak compactness and relaxation estimates</b>	157
5.1	Controls from the relative entropy bound	158
5.2	Controls from the entropy dissipation bound	161
5.3	Relaxation towards thermodynamic equilibrium	164
5.3.1	Infinitesimal Maxwellians	168
5.3.2	Bulk velocity and temperature	170
5.4	Improved integrability in velocity	174
<b>6</b>	<b>Lower-order linear constraint equations and energy inequalities</b>	183
6.1	Macroscopic constraint equations for one species	183
6.2	Macroscopic constraint equations for two species, weak interactions	186
6.3	Energy inequalities	190
6.4	The limiting Maxwell equations	196
<b>7</b>	<b>Strong compactness and hypoellipticity</b>	199
7.1	Compactness with respect to $v$	200
7.1.1	Compactness of the gain term	200
7.1.2	Relative entropy, entropy dissipation and strong compactness	203
7.2	Compactness with respect to $x$	207
7.2.1	Hypoellipticity and the transfer of compactness	208
7.2.2	Compactness of fluctuations for one species	213
7.2.3	Compactness of fluctuations for two species	221
<b>8</b>	<b>Higher-order and nonlinear constraint equations</b>	233
8.1	Macroscopic constraint equations for two species, weak interactions	233
8.1.1	Proof of Proposition 8.1	235
8.1.1.1	An admissible renormalization	235
8.1.1.2	Convergence of conservation defects	237
8.1.1.3	Decomposition of flux terms	242

8.1.1.4	Decomposition of acceleration terms . . . . .	244
8.1.1.5	Convergence . . . . .	247
8.2	Macroscopic constraint equations for two species, strong interactions . . . . .	247
8.3	Energy inequalities for two species, strong interaction . . . . .	258
<b>9</b>	<b>Approximate macroscopic equations . . . . .</b>	<b>263</b>
9.1	Approximate conservation of mass, momentum and energy for one species . . . . .	264
9.1.1	Conservation defects . . . . .	266
9.1.2	Decomposition of flux terms . . . . .	270
9.1.3	Decomposition of acceleration terms . . . . .	276
9.2	Approximate conservation of mass, momentum and energy for two species . . . . .	278
9.2.1	Conservation defects . . . . .	287
9.2.2	Decomposition of flux terms . . . . .	290
9.2.3	Decomposition of acceleration terms . . . . .	293
9.2.4	Proof of Propositions 9.5 and 9.6 . . . . .	295
9.2.5	Proofs of Lemmas 9.7, 9.8, 9.9 and 9.10 . . . . .	299
<b>10</b>	<b>Acoustic and electromagnetic waves . . . . .</b>	<b>313</b>
10.1	Formal filtering of oscillations . . . . .	314
10.2	Rigorous filtering of oscillations . . . . .	318
<b>11</b>	<b>Grad's moment method . . . . .</b>	<b>323</b>
11.1	Proof of Theorem 4.5 . . . . .	323
11.1.1	Weak convergence of fluctuations, collision integrands and electromagnetic fields . . . . .	323
11.1.2	Constraint equations, Maxwell's system and the energy inequality . . . . .	324
11.1.3	Evolution equations . . . . .	325
11.1.4	Temporal continuity, initial data and conclusion of the proof . . . . .	328
<b>12</b>	<b>The renormalized relative entropy method . . . . .</b>	<b>331</b>
12.1	The relative entropy method: old and new . . . . .	331
12.2	Proof of Theorem 4.6 on weak interactions . . . . .	332
12.2.1	Weak convergence of fluctuations, collision integrands and electromagnetic fields . . . . .	332
12.2.2	Constraint equations, Maxwell's system and the energy inequality . . . . .	334
12.2.3	The renormalized modulated entropy inequality . . . . .	335
12.2.4	Convergence and conclusion of the proof . . . . .	355
12.3	Proof of Theorem 4.7 on strong interactions . . . . .	356
12.3.1	Weak convergence of fluctuations, collision integrands and electromagnetic fields . . . . .	356

12.3.2 Constraint equations, Maxwell's system and the energy inequality . . . . .	359
12.3.3 The renormalized modulated entropy inequality . . . . .	360
12.3.4 Convergence and conclusion of the proof . . . . .	380
<b>Appendix A: Cross-section for momentum and energy transfer . . . . .</b>	<b>385</b>
<b>Appendix B: Young inequalities . . . . .</b>	<b>389</b>
<b>Appendix C: End of proof of Lemma 7.7 on hypoelliptic transfer of compactness . . . . .</b>	<b>393</b>
<b>Bibliography . . . . .</b>	<b>399</b>
<b>Index . . . . .</b>	<b>403</b>