Preface to the second edition

The present book is a new, substantially enlarged, version of a previous set of lecture notes, published first in 2008, and then in revised form a few years later, by the Edizioni della Scuola Normale.

The volume originally arose from notes taken during courses in classical diophantine analysis delivered at the Scuola Normale between 2006 and 2008. The main structure has been maintained in the present version, in that for instance there are still five chapters (plus an appendix by Francesco Amoroso); also, the style has not changed and generally the exposition has been kept at an elementary (and essentially self-contained) level, giving emphasis to some main ideas rather than to the finest technical results which can be obtained by the methods.

However, the present new edition contains very substantial additions, and expansions of the former content, so that the final shape is very far from the former. Many results have been added, in the form of new sections, new supplements, or new exercises. The arguments and proofs for such exercises are developed in full by means of “Hints”, which in fact are much more than scattered suggestions, and in practice contain complete detail. In order to highlight the content of these exercises, the goals are often expressed with titles in boldface, so that the interested reader can quickly see what is treated. Occasionally, the exercises contain results which have not been explicitly published.

To be more explicit, here is a list of some main ones among several new pieces:

Chapter 1: A whole section on (the basics of) reduction theory for binary integral quadratic forms has been added, together with some effective algorithms and applications; there is also a subsection treating, effectively, a certain indefinite (anisotropic) ternary form (whereas ternaries did not appear before). Also, a supplement on basic geometry of numbers now appears, with several results on successive minima. Further, the former supplement on Padé approximations to the exponential function, and its specializations to values at integers, has been considerably expanded.

Chapter 2: The former supplement on Runge’s theorem has been hugely expanded, with several extensions and improvements of the main results, and several new applications. Also, the chapter now contains a completely new series of exercises, for instance on special diophantine equations, and on the well-known function-field elliptic equation expressing representations of a polynomial as a difference between a square and a cube.

Chapter 3: Several facts related to heights have been inserted, for instance a self-contained simple proof of a lower bound (essentially by Blanksby–Montgomery) for the maximal conjugate of an algebraic integer, and also a proof of a result by Siegel about comparison of heights of values of algebraic functions (which appeared only in the rather easier case of rational functions). Further, the supplements now contain a proof of a result by Silverman–Tate about bounded height for torsion values of sections on an elliptic scheme over a curve; this is developed by keeping the prerequisites to a minimum.

Chapter 4: Here, in particular, we have added many results (often in the form of hinted exercises) concerning zeros of linear recurrence sequences over various fields, and concerning the (previously existent) Skolem–Mahler–Lech theorem. Also, there is a new short supplement with an (apparently) novel proof of a well-known theorem by Burnside on torsion matrix groups, with an argument which seems to be absent from the existing literature.

Chapter 5: We have added several further applications of the results on Padé approximations proved in the chapter, to heights and diophantine equations, which were absent from the previous versions of these notes. In particular, we have treated explicitly a certain family of cubic Thue equations, with conclusions which give effective estimates for diophantine approximations to certain cubic roots, of strength near to Roth's; one exercise treats this family by means of the Skolem method (explained in the previous chapter). Further, we have worked out quantitative applications to effective diophantine inequalities, by means of a method which is the object of work in progress (jointly with F. Amoroso and D. Masser).

We have naturally added a number of references, although this is without any aim to be complete (taking into account the nature of these notes). The appendix too has been revised and partially updated by Amoroso, whom I warmly thank.

In fact, as said, many more new results than mentioned above have been included, often in the shape of hinted exercises, of which a complete list would be too long.

We hope that the several new arguments of varied shape which appear, especially in the mentioned new parts, may be instructive, giving a rather more ample overview of the topic and its mathematical links, compared to the previous versions of the book.

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On the occasion of this revised edition, I add thanks to Gabriella Böhm for important help, and to the copyeditor Alison Durham for a very accurate analysis in several directions, and preparation of the final text, which also helped much in improving the presentation.