

For my family

Preface

The origins of this book are in a Part III lecture course I gave in the early 2020s in Cambridge. The course was devoted to profinite groups but also included material on group cohomology, with which profinite groups are intimately linked. The written notes for this course persisted on the internet, and enough new students of the topic have approached me to comment on the usefulness of these notes to convince me that there is a need for a new introductory work on the theory of profinite groups.

My initial intent was merely to revise and tidy up those course notes to make a reasonably short introduction to profinite groups and group cohomology. This introduction would provide a new student with a good grounding before they progressed to reading research papers or one of the more encyclopaedic works available, such as L. Ribes' early 1970 work *Introduction to Profinite Groups and Galois Cohomology* [114], J. Wilson's 1998 book *Profinite Groups* [142] or the 2010 tome of the same name by L. Ribes and P. Zalesskii [117]. However, in the course of writing, I came to realize that there are important areas of present interest not covered in the existing textbooks. In particular, my own favoured area of research – the connections of profinite groups to abstract groups via the profinite completion – is largely absent from the currently published books. Thus, I have expanded the plans of the book to include much more material on abstract groups and residual finiteness, and the linkages between these groups and the world of profinite groups. Each of the 'original' chapters has needed considerable expansion, and several entirely new ones have been added.

As a consequence of the focus on residually finite groups, the reader will find that this book has a considerably more 'geometric' flavour than most works on the subject. The examples especially are drawn from 'geometric' constructions of free groups, amalgams, and so on, as opposed to the more usual number-theoretic constructions of profinite groups. In particular I will say almost nothing including the name of Galois, for these areas of profinite mathematics have been treated far more fully elsewhere than I could hope to do.

This book is intended to be aimed at graduate students, at the early doctoral or magisterial stage of their studies. As such, I have attempted to keep assumed knowledge to a minimum as far as possible. At times the reader will be assumed to have an acquaintance with certain aspects of algebraic topology, in particular covering spaces, fundamental groups and simplicial homology. It is quite possible to learn about profinite groups without these topics, but to provide examples of groups with good residual properties the geometric connection is essential.

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