Basic notation and definitions

In what follows we will use the standard notation $\mathbb{C}$, $\mathbb{R}$, $\mathbb{Z}$ and $\mathbb{N}$, for sets of complex, real, integer and positive integer numbers, respectively. Moreover, let $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

For two points $z_1, z_2 \in \mathbb{C}$ we denote by $[z_1, z_2]$ the straight line segment from z_1 to z_2. Furthermore, we denote by $D(a,r)$ the open disc with center at some point $a \in \mathbb{C}$ and radius $r > 0$ so that $D(a,r) = \{z : |z-a| < r\}$. We also put $\mathbb{D} := D(0,1)$ and $\mathbb{T} := \{z : |z| = 1\}$ so that $\mathbb{D}$ and $\mathbb{T}$ are the open unit disk and the unit circle in $\mathbb{C}$, respectively.

We will denote by $\jmath$ the function such that $\jmath(z) = z$ without paying attention to its domain of definition that will be always clear from the context upon every appearance of the function $\jmath$.

Let $\mathrm{Area}(E)$ stand for the area of a measurable set $E \subset \mathbb{C}$, while the integral of a measurable function f over a measurable set E against the planar Lebesgue measure will be denoted by $\int_E f(z)\, dA(z)$.

Let us also denote by $m_{\mathbb{T}}$ the normalized Lebesgue measure on $\mathbb{T}$, so that for $\zeta = e^{i\vartheta}$ one has $dm_{\mathbb{T}}(\zeta) = \frac{1}{2\pi} d\vartheta$. Accordingly, for a rectifiable curve γ in $\mathbb{C}$ defined on a closed interval I one writes $dz|_\gamma$ (or dz if there are no doubts what γ one deals with) for the measure on $\gamma^* := \gamma(I)$ which acts (as a functional in the space of continuous complex valued functions on γ) by the formula $f \mapsto \int_\gamma f(z)\, dz$.

We will use the usual abbreviations a.a. and a.e. for the sentences "almost all" and "almost everywhere". In all cases when the corresponding measure is not mentioned explicitly, it will be completely clear from the context.

For a set $E \subset \mathbb{C}$ we will denote by $\overline{E}$, ∂E, $\mathrm{Int}(E) = E^\circ$, and $E^\complement = \mathbb{C} \setminus E$ the closure, the boundary, the interior, and the complement of E, respectively (all these topological entities are considered with respect to the topology of $\mathbb{C}$). As usual, a domain will mean a connected open set.

Let $\mathbb{C}_\infty$ stand for the standard one-point compactification of $\mathbb{C}$. The boundary of a set E in the topology of $\mathbb{C}_\infty$ will be denoted by $\partial_\infty E$.

If E is a subset of $\mathbb{C}$, we say that E *separates the plane* if the set $\mathbb{C} \setminus E$ is not connected. In what follows the word *component* will always mean *connected component*.

Let us recall, that a *Jordan curve* $\varGamma$ (in the literature it is called sometimes a simple closed curve) is a homeomorphic image of the unit circle $\mathbb{T}$. By virtue of the classical Jordan curve theorem (see, for instance, [19, page 102]) the set $\varGamma^\complement$ is not connected. It consists of two components. The bounded component of the set $\varGamma^\complement$ is called a *domain bounded by* $\varGamma$ and it will be denoted by $D(\varGamma)$. The unbounded component of $\varGamma^\complement$ will be denoted by $\Omega_\infty(\varGamma)$. It also follows from the Jordan curve theorem that $\varGamma = \partial D(\varGamma) = \partial \Omega_\infty(\varGamma)$.

By an *arc*, we will mean a homeomorphic image of $[0, 1]$.

For a function (or a measure) f and for a set E we will denote by $f|_E$ the restriction of f to E. For a set of functions $\mathcal{F}$ we will put $\mathcal{F}|_E = \{f|_E : f \in \mathcal{F}\}$.

In what follows we will denote by (a_n) a sequence (of objects a_n of any nature), and the index n will run over $\mathbb{N}_0$ or $\mathbb{N}$. The convergence of any sequence (a_n) will be always considered as $n \to \infty$.

Also we need to introduce several sets and spaces of functions which will be used in what follows. We will denote by $\mathcal{P}$ the set of polynomials in a complex variable, and by $\mathcal{R}$ the set of all complex rational functions defined on $\mathbb{C}_\infty$. The set of all poles of a given function $g \in \mathcal{R}$ will be denoted by $\{g\}_\infty$.

For a closed set $X \subset \mathbb{C}$ let $C(X)$ be the space of all bounded and continuous complex functions on X. This space is a Banach space with respect to the standard uniform norm $\|\cdot\|_X$, which is defined for $f \in C(X)$ as follows: $\|f\|_X = \sup_{z \in X} |f(z)|$. The space of all real-valued functions from $C(X)$ will be denoted by $C(X, \mathbb{R})$.

Let (g_n) be some sequence of functions defined on a compact set $X \subset \mathbb{C}$. Occasionally (in some bulky sentences) we will write $g_n \rightrightarrows g$ on X in order to say that this sequence converges uniformly on X to g, but more often we will write such a fact in a more traditional way, i.e., in the form "$g_n \to g$ uniformly on X". Let now (g_n) be a sequence of functions defined on an open set U. We will say that g_n converges to some function g *locally uniformly in* U, if for each compact subset $K \subset U$ we have $g_n \to g$ uniformly on K. This fact will be denoted as "$g_n \to g$ locally uniformly in U", or, in the short form, as "$g_n \rightrightarrows g$ locally in U".

For an open set Ω let $H(\Omega)$ and $H^\infty(\Omega)$ be the spaces of all *holomorphic* and *bounded holomorphic* functions in Ω, respectively. For a function $f \in H^\infty(\Omega)$ we put $\|f\|_{\infty,\Omega} = \|f\|_\Omega := \sup_{z \in \Omega} |f(z)|$. In what follows the space $H^\infty(\mathbb{D})$ will be simply denoted by H^∞, and the norm $\|\cdot\|_{\infty,\mathbb{D}}$ will be denoted by $\|\cdot\|_\infty$. For a compact set K let $H(K)$ be the space of restrictions to K of functions in $H(G)$, where G is some open set that contains K.

Furthermore, the symbol $\mathrm{Har}(\Omega)$ will denote the space of all *harmonic* functions in Ω. In different contexts we will consider real or complex valued harmonic functions, and in all cases when it will not be clear which class of functions we are dealing with we will use the more accurate notation $\mathrm{Har}(\Omega, \mathbb{R})$ and $\mathrm{Har}(\Omega, \mathbb{C})$ for the respective classes of harmonic functions.