

## References

- [1] E. Abakumov and K. Y. Fedorovskiy, **Analytic balayage of measures, Carathéodory domains, and badly approximable functions in  $L^p$ .** *C. R. Math. Acad. Sci. Paris* **356** (2018), no. 8, 870–874
- [2] A. V. Abanin and L. H. Khoi, **Cauchy transformation and mutual dualities between  $A^{-\infty}(\Omega)$  and  $A^\infty(\Omega^C)$  for Carathéodory domains.** *Bull. Belg. Math. Soc. Simon Stevin* **23** (2016), no. 1, 87–102
- [3] F. G. Abdullaev and A. A. Dovgoshei, **Szegő theorem, Carathéodory domains and boundedness of calculating functionals.** *Math. Notes* **77** (2005), no. 1, 3–14
- [4] J. T. Anderson, J. A. Cima, N. Levenberg, and T. J. Ransford, **Projective hulls and characterizations of meromorphic functions.** *Indiana Univ. Math. J.* **61** (2012), no. 6, 2111–2122
- [5] N. U. Arakeljan, Uniform approximation on closed sets by entire functions. *Izv. Akad. Nauk SSSR Ser. Mat.* **28** (1964), 1187–1206
- [6] S. Axler, **Harmonic functions from a complex analysis viewpoint.** *Amer. Math. Monthly* **93** (1986), no. 4, 246–258
- [7] S. Banach, *Théorie des opérations linéaires.* Chelsea, New York, 1955
- [8] A. D. Baranov, J. J. Carmona, and K. Y. Fedorovskiy, **Density of certain polynomial modules.** *J. Approx. Theory* **206** (2016), 1–16
- [9] A. D. Baranov and K. Y. Fedorovskiy, **Boundary regularity of Nevanlinna domains and univalent functions in model subspaces.** *Sb. Math.* **202** (2011), no. 12, 1723–1740
- [10] A. D. Baranov and K. Y. Fedorovskiy, **On  $L^1$ -estimates of derivatives of univalent rational functions.** *J. Anal. Math.* **132** (2017), 63–80
- [11] S. Y. Belov and K. Y. Fedorovskiy, **Model spaces containing univalent functions.** *Russ. Math. Surveys* **73** (2018), no. 1, 172–174
- [12] Y. Belov, A. Borichev, and K. Y. Fedorovskiy, **Nevanlinna domains with large boundaries.** *J. Funct. Anal.* **277** (2019), no. 8, 2617–2643
- [13] E. Bishop, **Measures orthogonal to polynomials.** *Proc. Natl. Acad. Sci. USA* **44** (1958), no. 3, 278–280
- [14] E. Bishop, **The structure of certain measures.** *Duke Math. J.* **25** (1958), 283–289
- [15] E. Bishop, **Boundary measures of analytic differentials.** *Duke Math. J.* **27** (1960), 331–340
- [16] A. Boivin, P. M. Gauthier, and P. V. Paramonov, **On uniform approximation by  $n$ -analytic functions on closed sets in  $\mathbb{C}$ .** *Izv. Math.* **68** (2004), no. 3, 447–459
- [17] P. S. Bourdon, **Density of the polynomials in Bergman spaces.** *Pacific J. Math.* **130** (1987), no. 2, 215–221
- [18] S. Browder, *Introduction to function algebras.* W. A. Benjamin, New York-Amsterdam, 1969

- [19] R. B. Burckel, *An introduction to classical complex analysis. Vol. 1*. Pure Appl. Math. 82, Academic Press, Harcourt Brace Jovanovich, New York-London, 1979
- [20] C. Carathéodory, Untersuchungen über die konformen Abbildungen von festen und veränderlichen Gebieten. *Math. Ann.* **72** (1912), no. 1, 107–144
- [21] C. Carathéodory, Über die Begrenzung einfach zusammenhängender Gebiete. *Math. Ann.* **73** (1913), no. 3, 323–370
- [22] C. Carathéodory, Über die gegenseitige Beziehung der Ränder bei der konformen Abbildung des Inneren einer Jordanschen Kurve auf einen Kreis. *Math. Ann.* **73** (1913), no. 2, 305–320
- [23] T. Carleman, Über die Approximation analytischer Funktionen durch lineare Aggregate von vorgegebenen Potenzen. *Arkiv. Mat. Astron. Fys.* **17** (1922), no. 9, 1–30
- [24] L. Carleson, Mergelyan's theorem on uniform polynomial approximation. *Math. Scand.* **15** (1964), 167–175
- [25] J. J. Carmona, Mergelyan's approximation theorem for rational modules. *J. Approx. Theory* **44** (1985), no. 2, 113–126
- [26] J. J. Carmona and K. Y. Fedorovskiy, Conformal maps and uniform approximation by polyanalytic functions. In *Selected topics in complex analysis*, pp. 109–130, Oper. Theory Adv. Appl. 158, Birkhäuser, Basel, 2005
- [27] J. J. Carmona and K. Y. Fedorovskiy, New conditions for uniform approximation by polyanalytic polynomials. *Proc. Steklov Inst. Math.* **279** (2012), 215–229
- [28] J. J. Carmona, K. Y. Fedorovskiy, and P. V. Paramonov, On uniform approximation by polyanalytic polynomials and the Dirichlet problem for bianalytic functions. *Sb. Math.* **193** (2002), no. 10, 1469–1492
- [29] J. J. Carmona and C. Pommerenke, On the argument oscillation of conformal maps. *Michigan Math. J.* **46** (1999), no. 2, 297–312
- [30] J. J. Carmona and C. Pommerenke, Decomposition of continua and prime ends. *Comput. Methods Funct. Theory* **3** (2003), no. 1–2, 253–272
- [31] J. G. Caughran, Polynomial approximation and spectral properties of composition operators on  $H^2$ . *Indiana Univ. Math. J.* **21** (1971), 81–84
- [32] E. F. Collingwood and A. J. Lohwater, *The theory of cluster sets*. Cambridge Tracts in Math. Phys. 56, Cambridge University Press, Cambridge, 1966
- [33] J. B. Conway, *Functions of one complex variable. I*. 2nd edn., Grad. Texts in Math. 11, Springer, New York-Berlin, 1978
- [34] J. B. Conway, *The theory of subnormal operators*. Math. Surveys Monogr. 36, American Mathematical Society, Providence, RI, 1991
- [35] P. Davis, *The Schwarz function and its applications*. Carus Math. Monogr. 17, Mathematical Association of America, Buffalo, NY, 1974
- [36] J. Deny, Sur l'approximation des fonctions harmoniques. *Bull. Soc. Math. France* **73** (1945), 71–73
- [37] A. A. Dovgoshei, Polynomial approximation of functions from Hardy classes in simply-connected domains in the plane. *Ukrainian Math. J.* **42** (1990), no. 9, 1126–1130

- [38] A. A. Dovgoshei, *The F. and M. Riesz' theorem and Carathéodory domains*. *Anal. Math.* **21** (1995), no. 3, 165–175
- [39] O. Dovgoshey, *Certain characterizations of Carathéodory domains*. *Comput. Methods Funct. Theory* **5** (2005), no. 2, 489–503
- [40] P. Duren, B. W. Romberg, and A. Shields, Linear functionals on  $H^p$  spaces with  $0 < p < 1$ . *J. Reine Angew. Math.* **238** (1969), 32–60
- [41] P. Duren and A. Schuster, *Bergman spaces*. Math. Surveys Monogr. 100, American Mathematical Society, Providence, RI, 2004
- [42] P. L. Duren, *Theory of  $H^p$  spaces*. Pure Appl. Math. 38, Academic Press, New York-London, 1970
- [43] L. C. Evans and R. F. Gariepy, *Measure theory and fine properties of functions*. Stud. Adv. Math., CRC Press, Boca Raton, FL, 1992
- [44] O. J. Farrell, *On approximation to a mapping function by polynomials*. *Amer. J. Math.* **54** (1932), no. 3, 571–578
- [45] O. J. Farrell, *On approximation to an analytic function by polynomials*. *Bull. Amer. Math. Soc.* **40** (1934), no. 12, 908–914
- [46] O. J. Farrell, *On approximation by polynomials to a function analytic in a simply connected region*. *Bull. Amer. Math. Soc.* **41** (1935), no. 10, 707–711
- [47] O. J. Farrell, *On the representation of bounded analytic functions by sequences of polynomials*. *Amer. J. Math.* **60** (1938), no. 3, 573–576
- [48] O. J. Farrell, *On approximation measured by a surface integral*. *SIAM J. Numer. Anal.* **3** (1966), 236–247
- [49] K. Y. Fedorovskiy, *Uniform  $n$ -analytic polynomial approximation of functions on rectifiable contours in  $\mathbb{C}$* . *Math. Notes* **59** (1996), no. 4, 435–439
- [50] K. Y. Fedorovskiy, *On some properties and examples of Nevanlinna domains*. *Proc. Steklov Inst. Math.* **253** (2006), 186–194
- [51] K. Y. Fedorovskiy, *Carathéodory domains and Rudin's converse of the maximum modulus principle*. *Sb. Math.* **206** (2015), no. 1, 161–174
- [52] K. Y. Fedorovskiy, *Carathéodory sets and analytic balayage of measures*. *Sb. Math.* **209** (2018), no. 9, 1376–1389
- [53] K. Y. Fedorovskiy and P. Paramonov, *On  $\text{Lip}^m$ -reflection of harmonic functions over boundaries of simple Carathéodory domains*. *Anal. Math. Phys.* **9** (2019), no. 3, 1031–1042
- [54] S. D. Fisher, *Function theory on planar domains*. Pure Math. Appl. (New York), John Wiley, New York, 1983
- [55] D. Gaier, *Lectures on complex approximation*. Birkhäuser, Boston, MA, 1987
- [56] T. W. Gamelin, *Uniform algebras*. Chelsea, New York, 1984
- [57] S. Gardiner, *Harmonic approximation*. London Math. Soc. Lecture Note Ser. 221, Cambridge University Press, Cambridge, 1995

- [58] J. Garnett, *Bounded analytic functions*. Pure Appl. Math. 96, Academic Press, Harcourt Brace Jovanovich, New York-London, 1981
- [59] J. B. Garnett and D. E. Marshall, *Harmonic measure*. New Math. Monogr. 2, Cambridge University Press, Cambridge, 2005
- [60] I. Glicksberg, *The abstract F. and M. Riesz theorem*. *J. Functional Analysis* **1** (1967), 109–122
- [61] G. M. Goluzin, *Geometric theory of functions of a complex variable*. Transl. Math. Monogr. 26, American Mathematical Society, Providence, RI, 1969
- [62] F. Hartogs and A. Rosenthal, *Über Folgen analytischer Funktionen*. *Math. Ann.* **104** (1931), no. 1, 606–610
- [63] V. P. Havin, Approximation in the mean by analytic functions. *Dokl. Akad. Nauk SSSR* **178** (1968), 1025–1028
- [64] L. I. Hedberg, *Weighted mean approximation in Carathéodory regions*. *Math. Scand.* **23** (1968), 113–122
- [65] L. I. Hedberg and T. H. Wolff, *Thin sets in nonlinear potential theory*. *Ann. Inst. Fourier (Grenoble)* **33** (1983), no. 4, 161–187
- [66] H. Hedenmalm, B. Korenblum, and K. Zhu, *Theory of Bergman spaces*. Grad. Texts in Math. 199, Springer, New York, 2000
- [67] L. L. Helms, *Introduction to potential theory*. Pure Appl. Math. XXII, Wiley-Interscience A division of John Wiley, New York-London-Sydney, 1969
- [68] J. G. Hocking and G. S. Young, *Topology*. Addison-Wesley, Reading, Mass.-London, 1961
- [69] K. Hoffman, *Banach spaces of analytic functions*. Dover, New York, 1988
- [70] G. P. Kapoor and A. Nautiyal, *Approximation of entire functions over Carathéodory domains*. *Bull. Austral. Math. Soc.* **25** (1982), no. 2, 221–229
- [71] M. V. Keldych, Sur l’approximation en moyenne quadratique des fonctions analytiques. *Rec. Math. [Mat. Sbornik] N.S.* **5** (1939), no. 47, 391–401
- [72] M. V. Keldych, Sur la représentation par des séries de polynômes des fonctions d’une variable complexe dans de domaines fermés. *Rec. Math. [Mat. Sbornik] N.S.* **8** (1940), no. 50, 137–148
- [73] M. V. Keldych, Sur l’approximation en moyenne par polynômes des fonctions d’une variable complexe. *Rec. Math. [Mat. Sbornik] N.S.* **16** (1945), no. 58, 1–20
- [74] D. Khavinson, *F. and M. Riesz theorem, analytic balayage, and problems in rational approximation*. *Constr. Approx.* **4** (1988), no. 4, 341–356
- [75] S. Ya. Khavinson, On some extremal problems of the theory of analytic functions. *Moskov. Gos. Univ. Učenye Zapiski Matematika* **148** (1951), no. 4, 133–143. English translation: Amer. Math. Soc. Transl. (2) **32** (1963), 139–154
- [76] J. Kim and W. Y. Lee, *Invariant subspaces for operators whose spectra are Carathéodory regions*. *J. Math. Anal. Appl.* **371** (2010), no. 1, 184–189

- [77] P. Koosis, *Introduction to  $H_p$  spaces*. 2nd edn., Cambridge Tracts in Math. 115, Cambridge University Press, Cambridge, 1998
- [78] K. Kuratowski, *Topology. Vol. II*. Academic Press, New York-London, 1968
- [79] M. A. Lavrentiev, *Sur les fonctions d'une variable complexe représentables par des séries de polynômes*. Hermann, Paris, 1936
- [80] H. Lebesgue, *Sur le problème de Dirichlet*. *Rend. Circ. Mat. di Palermo* **29** (1907), no. 1, 371–402
- [81] D. H. Luecking and L. A. Rubel, *Complex analysis*. Universitext, Springer, New York, 1984
- [82] F.-Y. Maeda and N. Suzuki, *The integrability of superharmonic functions on Lipschitz domains*. *Bull. London Math. Soc.* **21** (1989), no. 3, 270–278
- [83] A. I. Markouchevitch, Sur la représentation conforme des domaines à frontières variables. *Rec. Math. [Mat. Sbornik] N.S.* **1(43)** (1936), no. 6, 863–886
- [84] A. I. Markushevich, *Conformal mapping of regions with variable boundary and application to the approximation of analytic functions by polynomials*. Ph.D. thesis, Moscow, 1934
- [85] A. I. Markushevich, Theory of analytic functions. Vols. 1 and 2. *Prentice-Hall, Englewood Cliffs, NJ* (1965)
- [86] M. Y. Mazalov, *Example of a nonrectifiable Nevanlinna contour*. *St. Petersburg Math. J.* **27** (2016), no. 4, 625–630
- [87] M. Y. Mazalov, *On Nevanlinna domains with fractal boundaries*. *St. Petersburg Math. J.* **29** (2018), no. 5, 777–791
- [88] M. Y. Mazalov, P. V. Paramonov, and K. Y. Fedorovskiy, *Conditions for the  $C^m$ -approximability of functions by solutions of elliptic equations*. *Russ. Math. Surveys* **67** (2012), no. 6, 1023–1068
- [89] S. Mazurkiewicz, *Über erreichbare Punkte*. *Fund. Math.* **26** (1936), 150–155
- [90] S. N. Mergelyan, Uniform approximations of functions of a complex variable. *Uspehi Matem. Nauk (N.S.)* **7** (1952), no. 2, 31–122. English translation: Amer. Math. Soc. Transl. (2) **101** (1954)
- [91] S. N. Mergelyan, On completeness of systems of analytic functions. *Uspehi Matem. Nauk (N.S.)* **8** (1953), no. 4, 3–63. English translation: Amer. Math. Soc. Transl. (2) **19** (1962), 109–166
- [92] K. Miller, *Extremal barriers on cones with Phragmén–Lindelöf theorems and other applications*. *Ann. Mat. Pura Appl. (4)* **90** (1971), 297–329
- [93] Z. Nehari, *Conformal mapping*. Dover, New York, 1975
- [94] M. H. Newman, *Elements of the topology of plane sets of points*. Cambridge University Press, New York, 1964
- [95] A. G. O’Farrell, *A generalised Walsh–Lebesgue theorem*. *Proc. Roy. Soc. Edinburgh Sect. A* **73** (1975), 231–234

- [96] A. G. O'Farrell, Theorems of Walsh–Lebesgue type. In *Aspects of contemporary complex analysis (Proc. NATO Adv. Study Inst., Univ. Durham, Durham, 1979)*, edited by D. A. Brannan and J. G. Clunie, pp. 461–467, Academic Press, London–New York, 1980
- [97] A. G. O'Farrell, Five generalisations of the Weierstrass approximation theorem. *Proc. Roy. Irish Acad. Sect. A* **81** (1981), no. 1, 65–69
- [98] A. G. O'Farrell and F. Pérez-González, Pointwise bounded approximation by polynomials. *Math. Proc. Cambridge Philos. Soc.* **112** (1992), no. 1, 147–155
- [99] P. V. Paramonov,  $C^m$ -approximations by harmonic polynomials on compact sets in  $\mathbb{R}^n$ . *Russian Acad. Sci. Sb. Math.* **78** (1994), no. 1, 231–251
- [100] P. V. Paramonov,  $C^1$ -extension and  $C^1$ -reflection of subharmonic functions from Lyapunov–Dini domains into  $\mathbb{R}^N$ . *Sb. Math.* **199** (2008), no. 12, 1809–1846
- [101] P. V. Paramonov, On  $\text{Lip}^m$  and  $C^m$ -reflection of harmonic functions with respect to boundaries of Carathéodory domains in  $\mathbb{R}^2$ . *Vestn. Mosk. Gos. Tekh. Univ. im. N.E. Baumana, Estestv. Nauki [Herald of the Bauman Moscow State Tech. Univ., Nat. Sci.]* (2018), no. 4, 36–45
- [102] P. V. Paramonov and K. Y. Fedorovskiy, On  $C^m$ -reflection of harmonic functions and  $C^m$ -approximation by harmonic polynomials. *Sb. Math.* **211** (2020), no. 8, 1159–1170
- [103] C. Pommerenke, *Univalent functions*. Studia Math./Mathematische Lehrbücher XXV, Vandenhoeck & Ruprecht, Göttingen, 1973
- [104] C. Pommerenke, *Boundary behaviour of conformal maps*. Grundlehren Math. Wiss. 299, Springer, Berlin, 1992
- [105] I. I. Privalov, *Boundary properties of analytic functions*. 2nd edn., GITTL, Moscow, 1950. German translation: *Randeigenschaften analytischer Funktionen*, Hochschulbücher für Mathematik, Bd. 25. VEB Deutscher Verlag der Wissenschaften, Berlin, 1956
- [106] J. Z. Qiu, Density of polynomials. *Houston J. Math.* **21** (1995), no. 1, 109–118
- [107] T. Ransford, *Potential theory in the complex plane*. London Math. Soc. Stud. Texts 28, Cambridge University Press, Cambridge, 1995
- [108] R. C. Roan, Weak\* generators of  $H^\infty$  and  $l^1$ . *Pacific J. Math.* **71** (1977), no. 2, 537–544
- [109] R. C. Roan, Composition operators on  $H^p$  with dense range. *Indiana Univ. Math. J.* **27** (1978), no. 1, 159–162
- [110] A. Roth, Approximationseigenschaften und Strahlengrenzwerte meromorpher und ganzer Funktionen. *Comment. Math. Helv.* **11** (1938), no. 1, 77–125
- [111] L. A. Rubel and A. L. Shields, Bounded approximation by polynomials. *Bull. Amer. Math. Soc.* **69** (1963), 591–593
- [112] L. A. Rubel and A. L. Shields, Bounded approximation by polynomials. *Acta Math.* **112** (1964), 145–162
- [113] W. Rudin, Analyticity, and the maximum modulus principle. *Duke Math. J.* **20** (1953), no. 3, 449–457
- [114] W. Rudin, *Functional analysis*. Tata McGraw-Hill, New Delhi, 1977
- [115] W. Rudin, *Real and complex analysis*. 3rd edn., McGraw-Hill, New York, 1987

- [116] C. Runge, *Zur Theorie der Eindeutigen Analytischen Functionen*. *Acta Math.* **6** (1885), no. 1, 229–244
- [117] A. L. Šaginyan, On approximation in the mean by harmonic polynomials. *Akad. Nauk Armyan. SSR Dokl.* **19** (1954), 97–103
- [118] S. Saks and A. Zygmund, *Fonctions analytiques*. Masson et Cie, Éditeurs, Paris, 1970
- [119] D. Sarason, *Invariant subspaces and unstarred operator algebras*. *Pacific J. Math.* **17** (1966), 511–517
- [120] D. Sarason, Weak-star generators of  $H^\infty$ . *Pacific J. Math.* **17** (1966), 519–528
- [121] D. Sarason, On the order of a simply connected domain. *Michigan Math. J.* **15** (1968), 129–133
- [122] A. Shields, Carathéodory and conformal mapping. *Math. Intelligencer* **10** (1988), no. 1, 18–22
- [123] S. O. Sinanjan, Approximation by analytical functions and polynomials in the mean with respect to the area. *Mat. Sb. (N.S.)* **69** (1966), 546–578. English translation: Amer. Math. Soc. Transl. (2) **74** (1968), 91–124
- [124] V. I. Smirnov and N. A. Lebedev, *Functions of a complex variable: Constructive theory*. The MIT Press, Cambridge, MA, 1968
- [125] E. L. Stout, *The theory of uniform algebras*. Bogden & Quigley, Tarrytown-on-Hudson, NY, 1971
- [126] A. Stray, Pointwise bounded approximation by functions satisfying a side condition. *Pacific J. Math.* **51** (1974), 301–305
- [127] P. K. Suetin, Polynomials orthogonal over a region and Bieberbach polynomials. *Proc. Steklov Inst. Math.* **100** (1974), 1–91
- [128] A. G. Vitushkin, The analytic capacity of sets in problems of approximation theory. *Russian Math. Surveys* **22** (1967), no. 6, 139–200
- [129] J. L. Walsh, Über die Entwicklung einer analytischen Funktion nach Polynomen. *Math. Ann.* **96** (1927), no. 1, 430–436
- [130] J. L. Walsh, Über die Entwicklung einer Funktion einer komplexen Veränderlichen nach Polynomen. *Math. Ann.* **96** (1927), no. 1, 437–450
- [131] J. L. Walsh, On approximation to an arbitrary function of a complex variable by polynomials. *Trans. Amer. Math. Soc.* **30** (1928), no. 3, 472–482
- [132] J. L. Walsh, Über die Entwicklung einer harmonischen Funktion nach harmonischen Polynomen. *J. Reine Angew. Math.* **159** (1928), 197–209
- [133] J. L. Walsh, The approximation of harmonic functions by harmonic polynomials and by harmonic rational functions. *Bull. Amer. Math. Soc.* **35** (1929), no. 4, 499–544
- [134] J. L. Walsh, *Interpolation and approximation by rational functions in the complex domain*. Amer. Math. Soc. Colloq. Publ. XX, American Mathematical Society, Providence, RI, 1969
- [135] J. Wermer, On algebras of continuous functions. *Proc. Amer. Math. Soc.* **4** (1953), 866–869

- [136] G. T. Whyburn, *Analytic topology*. Amer. Math. Soc. Colloq. Publ. 28, American Mathematical Society, New York, 1971
- [137] T. Winiarski, [Approximation and interpolation of entire functions](#). *Ann. Polon. Math.* **23** (1970), 259–273