Preface

Since the seminal works of Gian-Carlo Rota and his school in the 1960s, combinatorics has become a respected branch of modern mathematics. Today it appears at the intersection of a wide variety of mathematical sciences. In fact, techniques and methods from combinatorics have become indispensable in such diverse areas as, for instance, statistical physics, number theory, and probability theory.

The work of Rota and his collaborators had a particularly profound impact on the development of algebraic combinatorics. As a result research on filtered bialgebras and Hopf algebras has seen a formidable expansion over the last two decades. Notably, some specific algebras, such as Faà di Bruno Hopf algebra, Rota–Baxter algebra, Grossman–Larson algebra, Malvenuto–Reutenauer Hopf algebra, and some pre-Lie algebras, among others, have appeared in different guises and in various and often seemingly unrelated domains. Such include renormalization in perturbative quantum field theory, Terry Lyons' rough path theory, dynamical systems, control theory and the analysis of numerical flows on manifolds.

The fundamental work of Alain Connes and Dirk Kreimer from the late 1990s plays a key role in the most recent developments. It resulted in an unforeseen flourishing of combinatorial, especially Hopf algebraic techniques, in theoretical physics, culminating in the work of Kreimer and his school on the so-called combinatorial Dyson–Schwinger equations.

Connes and Kreimer soon were advised on the value of Butcher's work on numerical integration methods from the early 1970s. In hindsight, Butcher's innovative use of rooted trees in the context of the analysis of Runge–Kutta methods was well ahead of its time. Today rooted trees naturally appear in many branches of pure and applied mathematics, and the Butcher group, together with the corresponding Hopf algebra of rooted trees, plays a paradigmatic role in the successful interaction between researchers working in the mathematical sciences.

As part of the French research network *GDR Renormalisation,*[1] the workshop *Dyson–Schwinger Equations and Faà di Bruno Hopf Algebras in Physics and Combinatorics* (*DSFdB2011*)[2] was hosted from June 27 to July 1, 2011 by IRMA at Strasbourg University in France. Its aim was to provide a platform which would allow researchers from different scientific communities, working around these themes, to exchange ideas and results.

We express our gratitude to all the speakers at DSFdB2011 and to the contributors of the present volume. We had the great pleasure to co-organize this meeting together with our colleagues and friends, Martin Bordemann from Mulhouse, France; Dorothea Bahns from Göttingen, Germany; and Dominique Manchon from Clermont–Ferrand, France. Special thanks go to Craig Roberts and W. Steven Gray for their efforts to

[1] http://renorm.math.cnrs.fr/

[2] http://www.th.physik.uni-bonn.de/people/fard/dsfdb2011/

convey some of the insights of a working physicist and a working engineer, respectively, to a mostly mathematical audience.

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For the present volume, experts have been invited to give comprehensive presentations of the connections between Dyson–Schwinger equations, Faà di Bruno algebras, and Butcher series. It is the result of the Strasbourg workshop, and the organizers hope it might serve as a timely reference for methods involving combinatorial Hopf algebras and related algebraic and combinatorial structures, and applications to physics, control theory, dynamical systems, analysis of numerical methods for differential equations, and beyond. It is exciting to see how research on these topics has continued to flourish since we met in Strasbourg, most notably, in the spectacular use of the same combinatorial methods in the theory of Lie–Butcher series, rough path theory, Fliess operators and stochastic differential equations. José M. Gracia–Bondía from Zaragoza, Spain, has written an insightful preface to this volume, as a commentary on the developments that have taken place over the last fifteen years or so. It helps to put these lectures into perspective. We are very grateful for his contribution.

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