Preface

The Workshop and International Conference on Representations of Algebras (ICRA 2012) took place in Bielefeld, Germany, during August 8–17, 2012. The aim of this book is to present some of the lectures to a wider audience.

The meeting consisted of a workshop (four days) with six lecture series and a conference (five days) with invited plenary lectures and contributed talks in seven parallel sessions; it attracted 268 mathematicians from 31 countries. The meeting was the 15th in a series of international conferences which started 1974 in Ottawa, and it marked a generation change with some new organisational structure, modernising the concept implemented by the first generation of representation theorists over the last 35 years.

The scientific organisers of the meeting were C. Bessenrodt, R. Farnsteiner, D. Happel, S. König, H. Krause, M. Reineke, J. Schröer. The external advisors were D. Benson, W. Crawley-Boevey, B. Keller, A. Kleshchev.

The ICRA Award 2012 (for outstanding work by young mathematicians in the field of representations of finite-dimensional algebras) was given to Jan Šťovíček for his key contributions to infinite-dimensional tilting theory and to approximation theory as well as for his work on representability of functors and existence of adjoints in triangulated categories.

This book contains ten expository survey articles. The participants of the meeting were not all specialists in the area and so the speakers aimed to make their talks as self-contained as possible. In fact, a characteristic feature of modern representation theory of algebras is the highly complex interaction with other branches of mathematics. Several of the powerful technologies developed within the field are contributing to other areas. In the opposite direction, new problems, ideas and points of view are coming into the subject from previously unrelated areas. These aspects are reflected in the papers presented here.

The study of derived categories and tilting is one of the most successful methodologies of algebra representation theory. Infinite-dimensional tilting modules are discussed in the paper of Angeleri Hügel. They occur when studying torsion pairs in module categories, when looking for complements to partial tilting modules, or in connection with the Homological Conjectures. They share many properties with classical tilting modules, but they also give rise to interesting new phenomena as they are intimately related with localisation, both at the level of module categories and of derived categories.

The last few years have seen the emergence of a new branch of representation theory for finite group schemes, namely modules of constant Jordan type. These are the analogues, among finitely generated modules, of vector bundles among coherent sheaves. Benson's article gives an introduction to this topic in the special case of elementary abelian p-groups, and explains how such modules give rise to vector bundles on projective space. The theory of Chern classes then feeds back information into the

representation theory. For the more general context of finite group schemes, see the article of Pevtsova.

The aim of the article by Bongartz is to survey old and new results in the theory of representation-finite and minimal representation-infinite algebras over an algebraically closed field. The cornerstones of the article are the existence of multiplicative bases and coverings with good properties, and applications of the ray-categories attached to distributive categories. The author presents several essential results of the classical representation theory of finite-dimensional algebras in a more general form and proposes simplified proofs for them. The article also contains several important applications of the presented theory: the sharper version of the second Brauer–Thrall conjecture, the author's recent result that there is no gap in the length of indecomposable modules of finite dimension over any algebra, the author's criterion for finite representation type, a new proof of the classification of representation-finite selfinjective algebras, as well as the theorem that every indecomposable module over a representation-finite algebra admits a basis such that all arrows in its quiver representation are represented by matrices having only 0 and 1 entries.

The paper by Brundan is a brief introduction to the quiver Hecke algebras discovered in 2008 by Khovanov, Lauda and Rouquier. They are certain Hecke algebras attached to symmetrisable Cartan matrices. It appears that Khovanov and Lauda came upon these algebras from an investigation of endomorphisms of Soergel bimodules (and related bimodules which arise from cohomology of partial flag varieties), while Rouquier's motivation was a close analysis of Lusztig's construction of canonical bases in terms of perverse sheaves on certain quiver varieties. The article explains these constructions in some detail, emphasising their application to the categorification of quantum groups.

The article of Brüstle and Yang gives a survey on ordered exchange graphs arising in cluster theory. The exchange graph of a cluster algebra encodes the combinatorics of mutations of clusters. Through the recent categorifications of cluster algebras using representation theory one obtains a whole variety of exchange graphs associated with objects such as a finite-dimensional algebra or a differential graded algebra concentrated in non-positive degrees. These constructions often come from variations of the concept of tilting, the vertices of the exchange graph being torsion pairs, t-structures, silting objects, support τ-tilting modules and so on. All these exchange graphs stemming from representation theory have the additional feature that they are the Hasse quiver of a partial order which is naturally defined for the objects. In this sense, the exchange graphs arising in cluster theory can be considered as a generalisation or as a completion of the poset of tilting modules which has been studied by Happel and Unger. The goal of the article is to axiomatise the thus obtained structure of an ordered exchange graph, to present the various constructions of ordered exchange graphs and to relate them among each other.

The aim of the lectures of Mozgovoy was to introduce refined Donaldson–Thomas invariants for the categories of modules over Jacobian algebras associated to quivers with potentials. These invariants were first studied in the context of 3-Calabi–Yau manifolds, but Kontsevich and Soibelman developed a framework that allows to shift the study of these invariants from 3-Calabi–Yau varieties to other sources of 3-Calabi–

Yau categories. One such source, closely related to representation theory, consists of quivers with potentials. The article follows closely these lectures. So the author defines refined Donaldson–Thomas invariants, computes them in some simple cases, and then discusses their basic properties, including integrality, positivity, and wall-crossing phenomena.

The article by Nakajima contains an approach to the Geiß–Leclerc–Schröer conjecture on the cluster algebra structure in the coordinate ring of a unipotent subgroup and the dual canonical base. This is related with the author's recent approach to the theory of cluster algebras based on perverse sheaves on graded quiver varieties. The new proposed idea is to use the singular support of a perverse sheaf, which is a lagrangian subvariety in the cotangent bundle of the space of quiver representations. The author surveys links between the related conjectures and approaches, and proposes two new conjectures which give links between the theory developed by Geiß, Leclerc and Schröer and perverse sheaves via singular support.

Pevtsova's article is an introduction to the representation theory and cohomology of finite group schemes, concentrating on the theory of π-points and Π-support, and leading into a discussion of constant Jordan type and related classes of modules in this general context. This nicely complements the article of Benson.

Superdecomposable pure-injective modules are discussed in the article by Prest. They form a particular class of infinite-dimensional modules and their existence reflects complexity in the category of finite-dimensional representations. For a finite-dimensional algebra, the evidence to date is consistent with existence of superdecomposable pure-injectives being equivalent to having non-domestic representation type. The paper describes this interplay between finite and infinite-dimensional modules in terms of pointed modules. Methods for producing superdecomposable pure-injectives are presented and some details are given in the context of tubular algebras.

A categorical framework for the theory of approximations and cotorsion pairs is presented in the paper by Šťovíček. His aim is to give a fairly complete account on the construction of compatible model structures on exact categories and symmetric monoidal exact categories, in some cases generalising previously known results. The discussion includes motivating applications with the emphasis on constructing monoidal model structures for the derived category of quasi-coherent sheaves of modules over a scheme.

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