

# Preface

This book is an outgrowth of the Impanga Conference on Algebraic Geometry, held at the Banach Center in Będlewo on July 4–10, 2010. For detailed information about this event, see

<http://www.impan.pl/~impanga/school>

The articles in this volume cover a broad range of topics in algebraic geometry: classical varieties, linear systems, birational geometry, Minimal Model Program, moduli spaces, toric varieties, enumerative theory of singularities, equivariant cohomology and arithmetic questions.

The book is based on contributions by conference speakers and participants, including two articles by mathematicians who were unable to attend the meeting. Here is a brief summary of the content of the volume.

The article by Klaus Altmann et al. describes the language of polyhedral divisors, which is used when working with  $T$ -varieties. This language is explained in parallel to the well established theory of toric varieties. In addition to basic constructions, subjects discussed include: singularities, separatedness and properness, divisors and intersection theory, cohomology, Cox rings, polarizations and equivariant deformations.

Dave Anderson gives an extensive introduction to equivariant cohomology in algebraic geometry. The first part is an overview, including basic definitions and examples. In the second part, the author discusses one of the most useful aspects of the theory: the possibility of localizing at fixed points without losing information. The third part focuses on Grassmannians, and describes some recent positivity results for their equivariant cohomology rings.

The article by Thomas Bauer et al. discusses open problems in the theory of linear systems. The discussion revolves around the speciality and the postulation problems as well as the containment problems for various powers of ideals. The main motivations come from the Harbourne–Hirschowitz, Nagata and the Bounded Negativity Conjectures.

Gergely Bérczi investigates the moduli space of holomorphic map germs from the complex line into complex compact manifolds. Two major applications are discussed: to Thom polynomials for Morin singularities in global singularity theory, and to the Green–Griffiths conjecture in the theory of hyperbolic algebraic varieties.

The article by Paolo Cascini and Vladmir Lazić gives an introduction to some recent developments in Mori theory aimed for a less experienced reader. In particular, the authors' recent direct proof of the finite generation of the canonical ring is discussed.

Sławomir Cynk and Sławomir Rams present various results on invariants of a resolution of a singular projective hypersurface. In particular, they prove some defect-type formulas. The proofs use, in an essential way, the sheaves of logarithmic differential forms.

Gavril Farkas studies the geometry of the moduli space of Prym varieties. Several applications of Prym varieties in algebraic geometry are presented. The exposition begins with a historical discussion of the life and achievements of Friedrich Prym. Topics treated in subsequent sections include singularities and Kodaira dimension of the moduli space, syzygies of Prym-canonical embedding and the geometry of the moduli space  $\mathcal{R}_g$  for small genus.

The article by Letterio Gatto and Inna Scherbak concerns Wrońskians in algebraic geometry. Wrońskians, usually introduced in standard courses in Ordinary Differential Equations (ODE), are a useful tool in algebraic geometry to detect ramification loci of linear systems. The authors describe some “materializations” of the Wrońskian, and of its close relatives, the *generalized Wrońskians* (labelled by partitions), in algebraic geometry. Emphasis is put on the relationship between Schubert calculus and ODE.

The article by Kevin Hitchinson and Masha Vlasenko supports Burt Totaro’s expectation that Bloch’s higher Chow groups of a field  $k$  can be computed using a very small class of affine algebraic varieties (linear spaces in the right coordinates), whereas the current definition uses essentially all algebraic cycles in affine space. The authors consider a simple modification of  $\mathrm{CH}^2(\mathrm{Spec}(k), 3)$  using only linear subvarieties in affine spaces, and show that it maps surjectively to the Bloch group  $B(k)$  for any infinite field  $k$ . They also describe the kernel of this map.

Andreas Hochenegger and Frederik Witt describe some constructions in symplectic toric geometry. The starting point is the Delzant construction of a symplectic toric manifold from a smooth polytope. The article then discusses the refinements for rational but not necessarily smooth polytopes due to Lerman and Tolman, leading to symplectic toric orbifolds or, more generally, symplectic toric DM stacks. The authors show that the latter stacks are isomorphic to the stacks obtained by Borisov et al. whenever the stacky fan is induced by a polytope.

Clemens Jörder and Stefan Kebekus address the following question. Let  $f : Y \rightarrow X$  be the inclusion map of a compact reduced subspace of a complex manifold, and let  $\mathcal{F} \subset T_X$  be a subsheaf of the tangent bundle which is closed under the Lie bracket, but not necessarily a sheaf of  $\mathcal{O}_X$ -algebras. When do infinitesimal deformations of  $f$  which are induced by  $\mathcal{F}$  lift to positive-dimensional deformations of  $f$ , where  $f$  is deformed along the sheaf  $\mathcal{F}$ ? In the case where  $X$  is complex-symplectic and  $\mathcal{F}$  is the sheaf of locally Hamiltonian vector fields, this partially reproduces known results on unobstructedness of deformations of Lagrangian submanifolds.

Michał Kapustka studies the variety of isotropic five-spaces of a degenerate four-form in a seven-dimensional vector space. It is a degeneration of the adjoint variety of the simple Lie group  $G_2$ . It is also the image of  $\mathbf{P}^5$  under the map induced by the system of quadrics containing a twisted cubic. Using degenerations of this twisted cubic to three lines, the author constructs geometric transitions between Calabi–Yau threefolds.

The article by Mateusz Michałek contains the notes from the lectures of Stefan Kebekus on hyperbolicity properties of moduli stacks and generalizations of Shafarevich hyperbolicity to higher dimensions. In the first part, various sheaves of differential forms on singular algebraic varieties are discussed. The second part collects some

facts on the Minimal Model Program and logarithmic sheaves. The last part is devoted mainly to hyperbolicity of moduli. It also discusses: pull-backs of differentials, a generalization of the Bogomolov–Sommese vanishing, and a special case of the Lipman–Zariski conjecture.

Mircea Mustață investigates an invariant of singularities, which plays an important role in birational geometry: the log canonical threshold. After its definition, properties and examples, the author describes an analogous invariant: the  $F$ -pure threshold, that comes up in positive characteristic in commutative algebra. Then a sketch of the proof of Shokurov’s ACC conjecture for ambient smooth varieties is presented, and a connection of this conjecture with Termination of Flips is described. The last part discusses an asymptotic version of the log canonical threshold in the context of graded sequences of ideals.

The article by Shigeru Mukai concerns K3 surfaces and Enriques surfaces. The author studies the connection between symplectic symmetries of K3 surfaces and the Mathieu group  $M_{24}$ . He also investigates its Enriques analogy, that is, a conjectural connection between semi-symplectic symmetries of Enriques surfaces and another Mathieu group  $M_{12}$ .

Özer Öztürk and Piotr Pragacz study Thom polynomials of singularities of maps, by expanding them in the basis of Schur functions (labelled by partitions). The article contains some necessary conditions on a partition to appear in the set of indices of the Schur function expansion of a Thom polynomial. Moreover, the authors describe several recursions for the coefficients of the Schur functions in these expansions. The article also shows old and new computations of the Thom polynomials of some singularities.

Marek Szyjewski investigates the kernel of the norm map on power classes for cyclic field extensions. In particular, he gives several results on the exactness of the “generalized Gross–Fischer exact sequence”.

Halszka Tutaj-Gasińska discusses a certain connection between the (local) positivity of line bundles on smooth projective varieties and the symplectic packing of balls into symplectic manifolds. The article collects some results on Seshadri constants and packing constants – two kinds of numerical invariants appearing in this connection. A special emphasis is put on the toric situation.

We dedicate the whole book to the memory of Oscar Zariski – the father of modern algebraic geometry. The opening article by Piotr Blass discusses his life and achievements. In particular, the influence of Zariski’s work and the work of his students on contemporary algebraic geometry is emphasized.

This volume completes the Impanga trilogy whose first two volumes are “Topics in cohomological studies of algebraic varieties” and “Algebraic cycles, sheaves, shtukas, and moduli”, published by Birkhäuser in 2005 and 2008, respectively.

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*The Editor*